

# Chapter 4:

## Column Distillation: Internal Stage-by-Stage Balances

In Chapter 3 (Introduction to Column Distillation), we performed the *external* balances or the balances around the distillation column

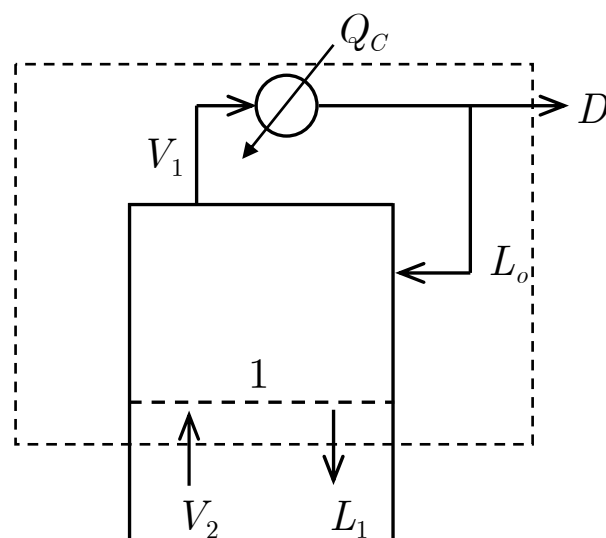
To obtain the answers for both design and specification problems, just doing the *external* balances is **NOT enough**, and, therefore, the *internal stage-by-stage* balances are **required**

We can perform the *internal stage-by-stage* balances from either the top (*i.e.* a condenser) or the bottom (*i.e.* a re-boiler) of the column

In the *enriching (rectifying)* section (*i.e.* the *section above the feed stage*), it is convenient to perform the balances for the area that goes around the desired stage and the condenser

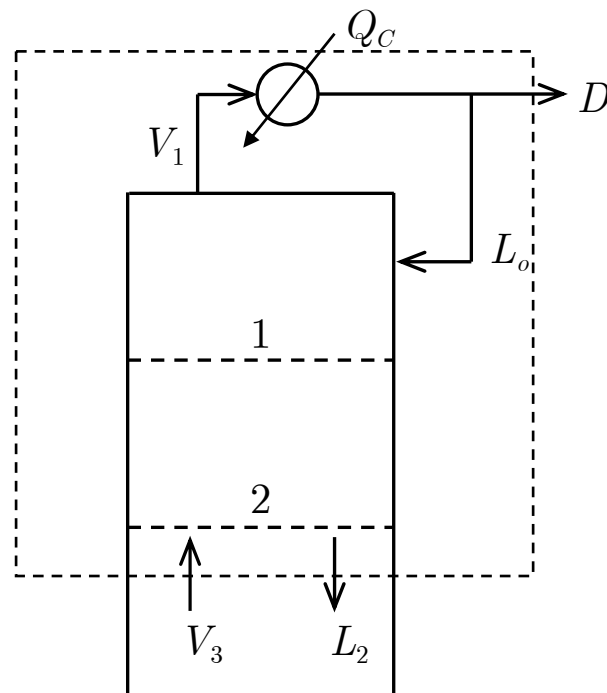
For stages 1, 2, and  $j$ , the balance envelopes (boundaries) can be drawn as follows

Stage 1



**Figure 4.1: An internal balance around stage 1**

## Stage 2

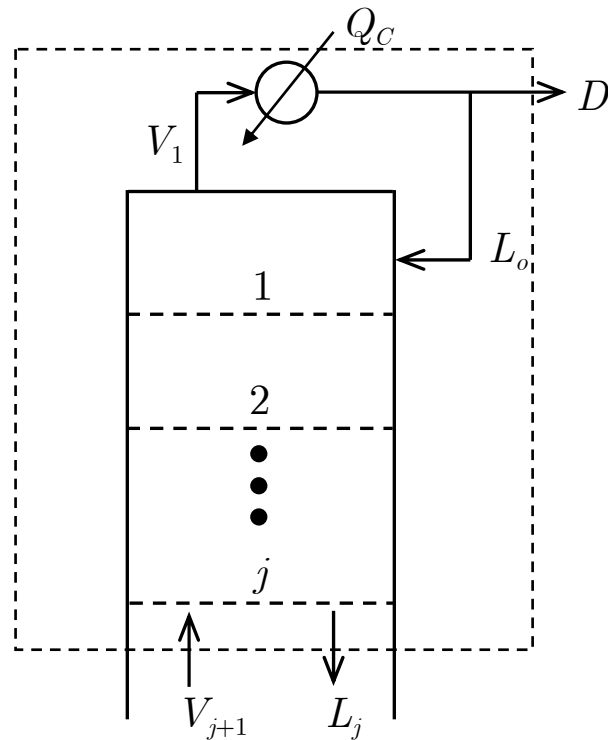


**Figure 4.2: An internal balance for stages 1 and 2**

Note that, in each stage, the *vapour* and the *liquid* phases are assumed to be *in equilibrium* with each other

Thus, in **each stage**, the **temperature** and **pressure** can be assumed to be **constant**

## Stage $j$



**Figure 4.3:** An internal balance for stages 1, 2, 3,..., and  $j$

The balances for stage 1 (Figure 4.1) can be written as follows

### Overall balance

$$V_2 = L_1 + D \quad (4.1)$$

Species balance (for a *more volatile* species)

$$y_2 V_2 = x_1 L_1 + x_D D \quad (4.2)$$

Energy balance

$$H_2 V_2 + Q_C = h_1 L_1 + h_D D \quad (4.3)$$

Note that, from now on, to avoid any confusion, an *enthalpy* of the **vapour phase** is denoted as **upper-case (capital letter)  $H$** , while the enthalpy of the *liquid phase* is still denoted as lower-case  $h$

It is also noteworthy that, in most of the distillation problems, the pressure ( $P$ ) of the system or the column is normally specified; thus the corresponding equilibrium data or curve can be obtained

If it is assumed that the **vapour phase** is a **saturated vapour** and that the **liquid phase** is a **saturated liquid**, we obtain the facts that

$$h_1 = h_1(x_1) \quad (4.4a)$$

and 
$$H_2 = H_2(y_2) \quad (4.4b)$$

Additionally, as it is assumed that the liquid and the vapour phases are in equilibrium with each other, the *mole fraction* of the *vapour* stream leaving stage 1 ( $y_1$ ) is *proportional* to that of the *liquid* stream leaving the same stage ( $x_1$ ) as follows

$$x_1 = x_1(y_1) \quad (4.4c)$$

or 
$$y_1 = y_1(x_1) \quad (4.4d)$$

For stage 2, the balance equations are as follows

$$V_3 = L_2 + D \quad (4.5)$$

$$y_3 V_3 = x_2 L_2 + x_D D \quad (4.6)$$

$$H_3 V_3 + Q_C = h_2 L_2 + h_D D \quad (4.7)$$

$$h_2 = h_2(x_2) \quad (4.8a)$$

$$H_3 = H_3(y_3) \quad (4.8b)$$

$$x_2 = x_2(y_2) \quad (4.8c)$$

Likewise, for stage  $j$ , the balance equations can generally be written as follows

$$V_{j+1} = L_j + D \quad (4.9)$$

$$y_{j+1} V_{j+1} = x_j L_j + x_D D \quad (4.10)$$

$$H_{j+1} V_{j+1} + Q_C = h_j L_j + h_D D \quad (4.11)$$

$$h_j = h_j(x_j) \quad (4.12a)$$

$$H_{j+1} = H_{j+1}(y_{j+1}) \quad (4.12b)$$

$$x_j = x_j(y_j) \quad (4.12c)$$

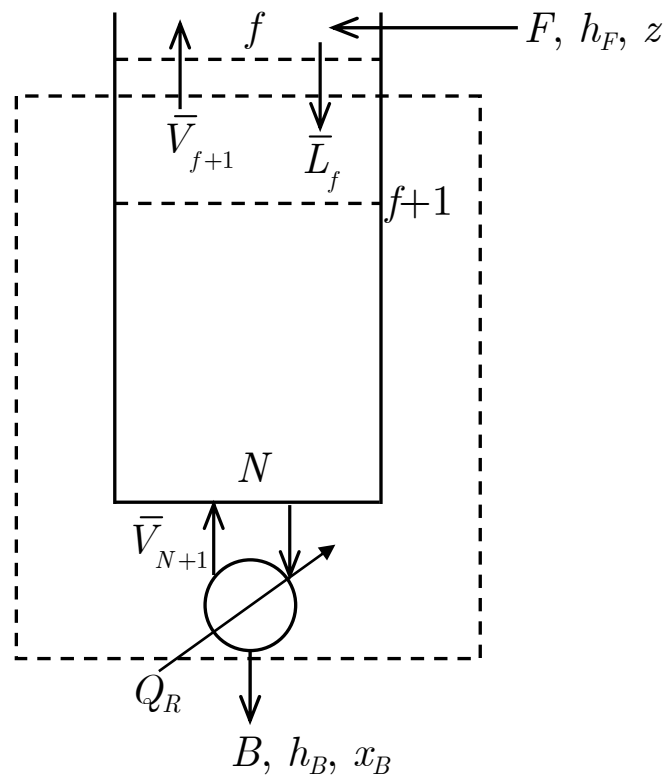
The *known* values (which were obtained previously from the *external* column balances – see the details in Chapter 3) are  $y_j$ ,  $D$ ,  $Q_C$ , and  $h_D$

Thus, there are **6 unknowns**; *i.e.*  $L_j$ ,  $V_{j+1}$ ,  $x_j$ ,  $y_{j+1}$ ,  $H_{j+1}$ , and  $h_j$

Since we have **6 equations**, we can solve for these 6 unknowns



For the *stripping* section or the section *below the feed stage*, the balances for the stage just below the feed plate (stage) or stage  $f+1$  can be depicted as follows (Figure 4.4)



**Figure 4.4: The internal balances for the stage just below the feeding stage (stage  $f + 1$ )**

From Figure 4.4, the balance equations can be written as follows

$$\bar{L}_f = \bar{V}_{f+1} + B$$

(note that, to distinguish between the flow rate **above** and **below** the feed stage, a *bar* is *added* to the flow rates of the liquid and the vapour phases *below the feed stage*)

or

$$\bar{V}_{f+1} = \bar{L}_f - B \quad (4.13)$$

$$y_{f+1} \bar{V}_{f+1} = x_f \bar{L}_f - x_B B \quad (4.14)$$

$$h_f \bar{L}_f + Q_R = H_{f+1} \bar{V}_{f+1} + h_B B$$

or

$$H_{f+1} \bar{V}_{f+1} = h_f \bar{L}_f - h_B B + Q_R \quad (4.15)$$

and

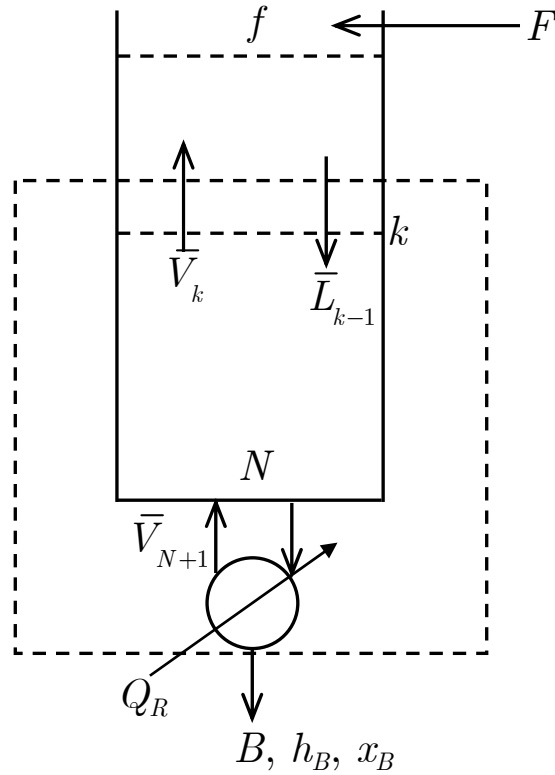
$$h_f = h_f(x_f) \quad (4.16a)$$

$$H_{f+1} = H_{f+1}(y_{f+1}) \quad (4.16b)$$

$$x_f = x_f(y_f) \quad (4.16c)$$

Again, there are **6 unknowns**, with **6 equations** available; thus, the problem can be solved

For stage  $k$ , which is the stage below the feed stage, the balances are carried out from the reboiler to the desired stage (*i.e.* stage  $k$ ), which can be illustrated in Figure 4.5



**Figure 4.5: The internal balances for a stage below the feed stage (*e.g.*, stage  $k$ )**

The balance equations for Figure 4.5 or the balance equations for a stage below the feed stage can be formulated as follows

$$\bar{V}_k = \bar{L}_{k-1} - B \quad (4.17)$$

$$y_k \bar{V}_k = x_{k-1} \bar{L}_{k-1} - x_B B \quad (4.18)$$

$$H_k \bar{V}_k = h_{k-1} \bar{L}_{k-1} - h_B B + Q_R \quad (4.19)$$

$$h_{k-1} = h_{k-1}(x_{k-1}) \quad (4.20a)$$

$$H_k = H_k(y_k) \quad (4.20b)$$

$$x_{k-1} = x_{k-1}(y_{k-1}) \quad (4.20c)$$

## 4.1 Stage-by-Stage Solution Methods for a Binary Mixture

As the stage-by-stage balances are rather complicated, before solving for the unknowns, the following **assumptions** are made:

- 1) The *column* is *adiabatic*
- 2) The *enthalpy changes* between the *adjacent stages* are **small**, compared to the *latent heat* changes (*i.e.* the changes in phase between the vapour and the liquid phases); *i.e.*

$$\left|H_{j+1} - H_j\right| \ll \lambda \text{ and } \left|h_{j+1} - h_j\right| \ll \lambda$$

3) The *latent heat* of vaporisation (or condensation) *per mole*,  $\lambda$ , is relatively *constant* throughout the column

By making the above *assumptions* (especially # 3), it yields the following facts:

$$L_1 \approx L_2 \approx L_3 \dots \approx L_j \dots \approx L_{f-1} \approx L \tag{4.21}$$

$$V_1 \approx V_2 \approx V_3 \dots \approx V_j \dots \approx V_{f-1} \approx V \tag{4.22}$$

$$\bar{L}_f \approx \bar{L}_{f+1} \dots \approx \bar{L}_{k-1} \approx \bar{L}_k \dots \approx \bar{L}_N = \bar{L} \tag{4.23}$$

$$\bar{V}_f \approx \bar{V}_{f+1} \dots \approx \bar{V}_{k-1} \approx \bar{V}_k \dots \approx \bar{V}_N \approx \bar{V} \tag{4.24}$$

However, it is important to note that

$$L \neq \bar{L}$$

and that

$$V \neq \bar{V}$$

This technique is developed by W.K. Lewis (1922) and called a *constant molal overflow* (**CMO**)

For stage  $j$  in the *enriching* section,

$$V_{j+1} = L_j + D \quad (4.9)$$

or

$$D = V_{j+1} - L_j \quad (4.25)$$

$$y_{j+1}V_{j+1} = x_jL_j + x_D D \quad (4.10)$$

Combining Eq. 4.10 with Eq. 4.25 yields

$$y_{j+1}V_{j+1} = x_jL_j + x_D (V_{j+1} - L_j) \quad (4.26)$$

Re-arranging Eq. 4.26 for  $y_{j+1}$  gives

$$y_{j+1} = \frac{L_j}{V_{j+1}} x_j + \left(1 - \frac{L_j}{V_{j+1}}\right) x_D \quad (4.27)$$

but, from Eqs. 4.21 and 4.22, for the *enriching* section,  $L$  and  $V$  are constant

Hence, Eq. 4.27 becomes

$$y_{j+1} = \frac{L}{V} x_j + \left(1 - \frac{L}{V}\right) x_D \quad (4.28)$$

**Eq. 4.28** is the *operating equation* for the *enriching section*

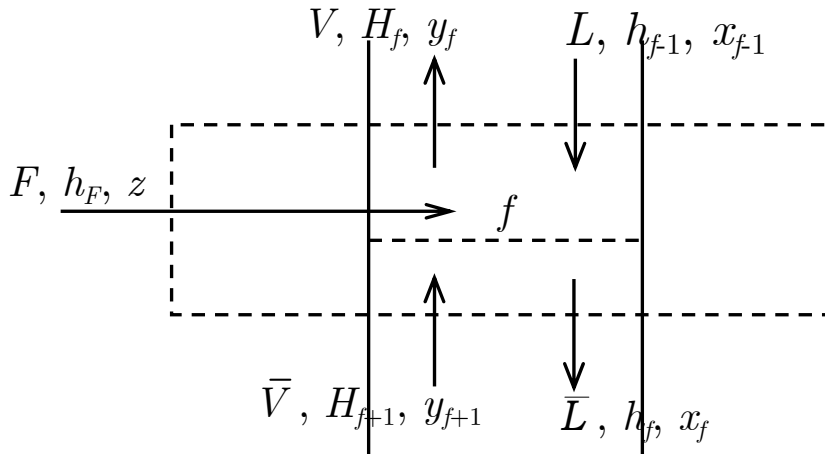
By doing the same for the *stripping* section, the following equation is obtained



$$y_k = \frac{\bar{L}}{\bar{V}} x_{k-1} - \left( \frac{\bar{L}}{\bar{V}} - 1 \right) x_B \quad (4.29)$$

**Eq. 4.29** is the *operating equation* for the *stripping section*

For the feed stage, the balances can be performed as follows (see the balance boundary in Figure 4.6)



**Figure 4.6:** The balances around the feed stage

$$F + \bar{V} + L = \bar{L} + V \quad (4.30)$$

$$h_F F + H_{f+1} \bar{V} + h_{f-1} L = h_f \bar{L} + H_f V \quad (4.31)$$

but, from the assumption # 2,  $H_{f+1} \approx H_f$  and

$$h_f \approx h_{f-1}$$

Thus, Eq. 4.31 becomes

$$h_F F + H \bar{V} + h L \approx h \bar{L} + H V$$

$$h_F F + H \bar{V} - H V \approx h \bar{L} - h L$$

and

$$h_f F + (\bar{V} - V) H \approx (\bar{L} - L) h \quad (4.32)$$

Re-arranging Eq. 4.30 yields

$$\bar{V} - V = \bar{L} - L - F \quad (4.33)$$

Substituting Eq. 4.33 into Eq. 4.32 gives

$$h_f F + H(\bar{L} - L - F) \approx h(\bar{L} - L) \quad (4.34)$$

Re-arranging Eq. 4.34 results in

$$h_f F + H(\bar{L} - L) - HF \approx h(\bar{L} - L)$$

and

$$(H - h)(\bar{L} - L) \approx (H - h_F)F \quad (4.35)$$

which can be re-arranged once again to

$$\frac{\bar{L} - L}{F} \approx \frac{H - h_F}{H - h} \quad (4.36a)$$

Note that  $\bar{L} - L$  is the amount of the *liquid* phase that goes down from the enriching section into the stripping section

Hence, the ratio  $\frac{\bar{L} - L}{F}$  is essentially  $q$  in the flash distillation (*i.e.* the *fraction* of the *feed* that becomes *liquid* and thus goes down to the bottom part of the tank); accordingly, Eq. 4.36a can also be written as

$$q = \frac{\bar{L} - L}{F} \approx \frac{H - h_F}{H - h} \quad (4.36b)$$

which can be in a narrative form as

$$q = \frac{\left[ \begin{array}{l} \text{Liquid flow rate} \\ \text{below the feed stage} \end{array} \right] - \left[ \begin{array}{l} \text{Liquid flow rate} \\ \text{above the feed stage} \end{array} \right]}{\text{Feed flow rate}} \quad (4.36c)$$

or

$$q = \frac{\left[ \begin{array}{l} \text{Enthalpy of the vapour} \\ \text{phase at the feed stage} \end{array} \right] - \left[ \begin{array}{l} \text{Enthalpy of the feed} \end{array} \right]}{\left[ \begin{array}{l} \text{Enthalpy of the vapour} \\ \text{phase at the feed stage} \end{array} \right] - \left[ \begin{array}{l} \text{Enthalpy of the liquid} \\ \text{phase at the feed stage} \end{array} \right]} \quad (4.36d)$$

Re-arranging Eq. 4.36b:

$$q = \frac{\bar{L} - L}{F}$$

gives

$$\bar{L} - L = qF \quad (4.37a)$$

or

$$\bar{L} = L + qF \quad (4.37b)$$

Combining Eq. 4.37a with Eq. 4.33:

$$\bar{V} - V = (\bar{L} - L) - F \quad (4.33)$$

and re-arranging the resulting equation yields

$$\begin{aligned} \bar{V} - V &= qF - F \\ \bar{V} &= V - (1 - q)F \end{aligned} \quad (4.38)$$

The use of the CMO technique can be illustrated in the following Example

**Example** A steady-state, counter-current, staged distillation column is used to separate ethanol (EtOH) from water

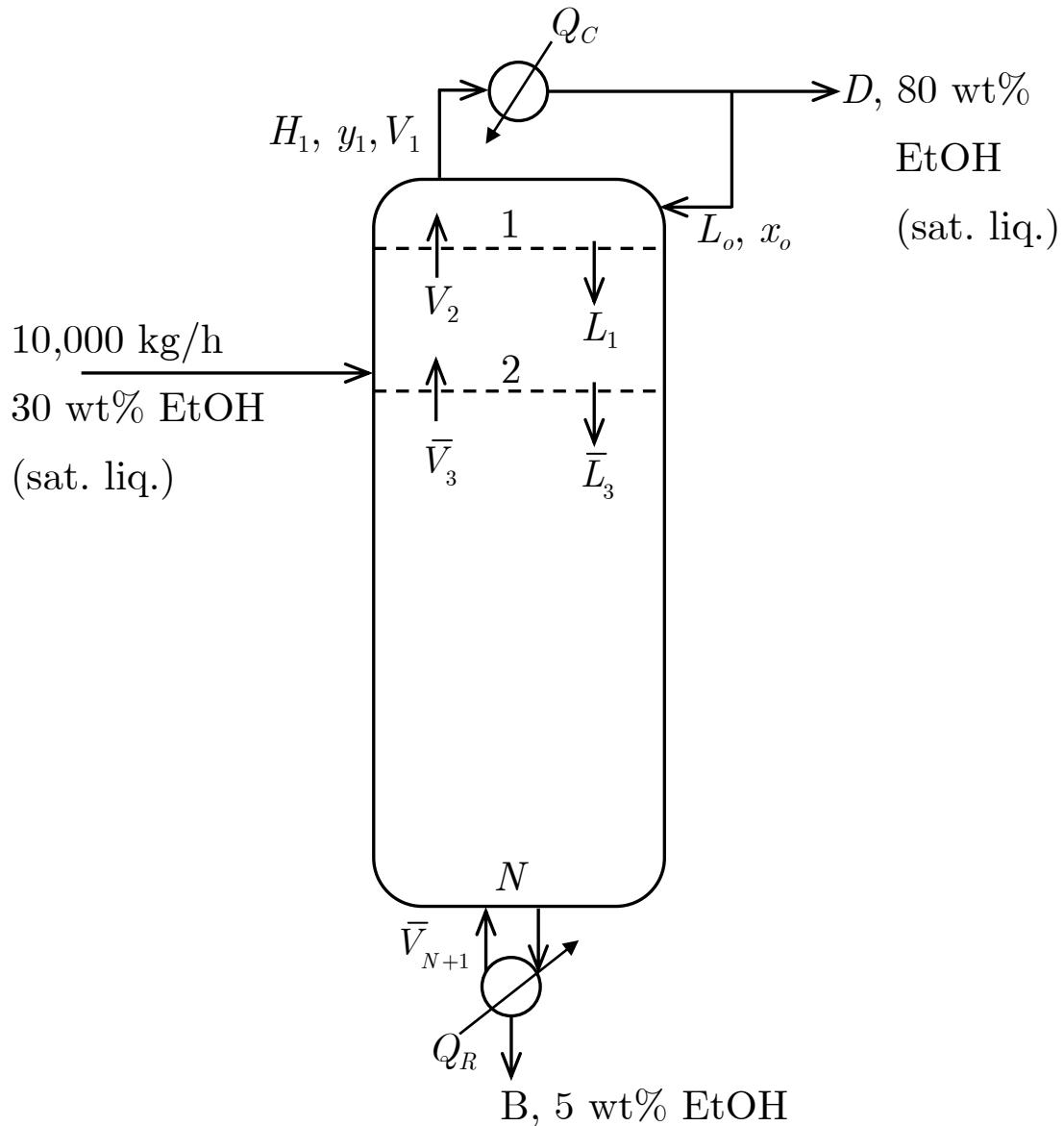
The *feed* is 30 wt% ethanol and is a *saturated liquid*

The mass flow rate of the feed is 10,000 kg/h, and the column is operated at the *atmospheric* pressure (*i.e.* 1 atm)

The *reflux* is assumed to be a *saturated liquid* and with the reflux ratio of 3.0

If the desired  $x_B$  and  $x_D$  are 0.05 (mass fraction of EtOH) and 0.80 (mass fraction of EtOH), respectively, use the Lewis method to find the number of stages required; given that the feed is fed on the 2<sup>nd</sup> stage from the top

The problem statement can be depicted as follows



From the Figure above, for EtOH,

- $z_F = 0.30$
- $x_D = x_o = y_1 = 0.80$
- $x_B = 0.05$

In order to utilise the Lewis method, all the assumptions (see Pages 13-14) must be satisfied, and the most important one is the assumption # 3 [*i.e.* the heat of vaporisation (or condensation) *per* mole is relatively constant]

Since we are dealing with 2 species, and the composition of the mixture varies from stage to stage, in order for the heat of vaporisation (or condensation) *per* mole to be relatively constant, the heats of vaporisation (or condensation) *per* mole of both components (*i.e.* EtOH and water in this Example) should be relatively close to each other



The latent heats of vaporisation of

- EtOH = 9.2 kcal/mol
- water = 9.7 kcal/mol

which are relatively close to each other

Thus, in this Example, it is safe to assume that the heat of vaporisation/condensation *per* mole of the mixture is nearly constant

Accordingly, the CMO (*constant molal overflow*) technique (or the Lewis method) can be employed to solve this Example

Hence, we have to deal with this Example on “*mole*” basis

Thus, all the given data, which are on “*mass*” basis, must be converted to “*mole*” basis

Note that the molecular weights (MW) of

- EtOH  $\approx 46$
- water  $\approx 18$

The mole fraction of EtOH of each stream can be computed as follows (note that the mole fraction is denoted with bar)

$$\bullet \bar{z}_F = \frac{\frac{0.30}{46}}{\frac{0.30}{46} + \frac{0.70}{18}} = 0.144$$

$$\bullet \bar{x}_D = \frac{\frac{0.80}{46}}{\frac{0.80}{46} + \frac{0.20}{18}} = 0.61$$

$$\bullet \bar{x}_B = \frac{\frac{0.05}{46}}{\frac{0.05}{46} + \frac{0.95}{18}} = 0.02$$

The feed flow rate is 10,000 kg/h, with the concentration of EtOH of 30 wt%

Thus, the *mass* flow rate of each component in the feed is

$$\bullet \text{EtOH} = \left(\frac{30}{100}\right)(10,000) = 3,000 \text{ kg/h}$$

$$\bullet \text{water} = \left(\frac{70}{100}\right)(10,000) = 7,000 \text{ kg/h}$$

which can be converted to the *molar* flow rate as follows

- EtOH:  $\frac{3,000}{46} = 65.2 \text{ kmol/h}$
- water:  $\frac{7,000}{18} = 388.9 \text{ kmol/h}$

Hence, in total, the *molar* flow rate of the feed is

$$65.2 + 388.9 = 454.1 \text{ kmol/h}$$

Since there is **no reaction** in the distillation column, we can perform a mole balance in the similar manner as we do for the mass balance, as follows

Overall mole balance

$$\begin{aligned}
 F &= D + B \\
 454.1 &= D + B
 \end{aligned}
 \tag{4.39}$$

## EtOH mole balance

$$\begin{aligned}\bar{z}_F F &= \bar{x}_D D + \bar{x}_B B \\ (0.144)(454.1) &= (0.61)D + (0.02)B\end{aligned}\tag{4.40}$$

Solving Eqs. 4.39 and 4.40 simultaneously for  $D$  and  $B$  yields

- $D = 95.3$  kmol/h
- $B = 358.8$  kmol/h

It is given that the reflux ratio  $\left(\frac{L_o}{D}\right)$  is 3.0

Thus,

$$\begin{aligned}L_o &= \left(\frac{L_o}{D}\right) \times D && (4.41) \\ L_o &= (3.0)(95.3) \\ L_o &= 285.9 \text{ kmol/h}\end{aligned}$$

Since the CMO technique is employed (Eq. 4.21),

$$L_o \approx L_1 \approx \dots L_{f-1} = L = 285.9 \text{ kmol/h}$$

By performing a mole balance around the condenser, we obtain

$$V_1 = L_o + D \quad (4.42)$$

$$V_1 = 285.9 + 95.3 = 381.2 \text{ kmol/h}$$

Once again, since the CMO technique is used (Eq. 4.22),

$$V_1 \approx V_2 \approx \dots V_f = V = 381.2 \text{ kmol/h}$$

Accordingly,

$$\begin{aligned} \frac{L}{V} \text{ (for the } \textit{rectifying} \text{ section)} &= \frac{285.9}{381.2} \\ &= 0.75 \end{aligned}$$

Consider the feed stage (*i.e.* the 2<sup>nd</sup> stage)

Since the **feed** is a *saturated liquid*, it does **not add** any flow rate to the *vapour phase* (**all** of the **feed** is **added** to the *liquid phase*)

Thus,

$$\bar{L}_2 = L_1 + F \quad (4.43)$$

$$\bar{L}_2 = 285.9 + 454.1 = 740.0 \text{ kmol/h}$$

With the CMO technique, we obtain the fact that (Eq. 4.23)

$$\bar{L}_2 \approx \bar{L}_3 \approx \bar{L}_4 \approx \dots \bar{L}_N = \bar{L} = 740.0 \text{ kmol/h}$$

Again, by applying the CMO technique, the flow rate of the vapour phase below the feed stage is constant: *i.e.* (Eq. 4.24)

$$V_2 = \bar{V}_3 \approx \bar{V}_4 \approx \dots \bar{V}_{N+1} = \bar{V} = 381.2 \text{ kmol/h}$$

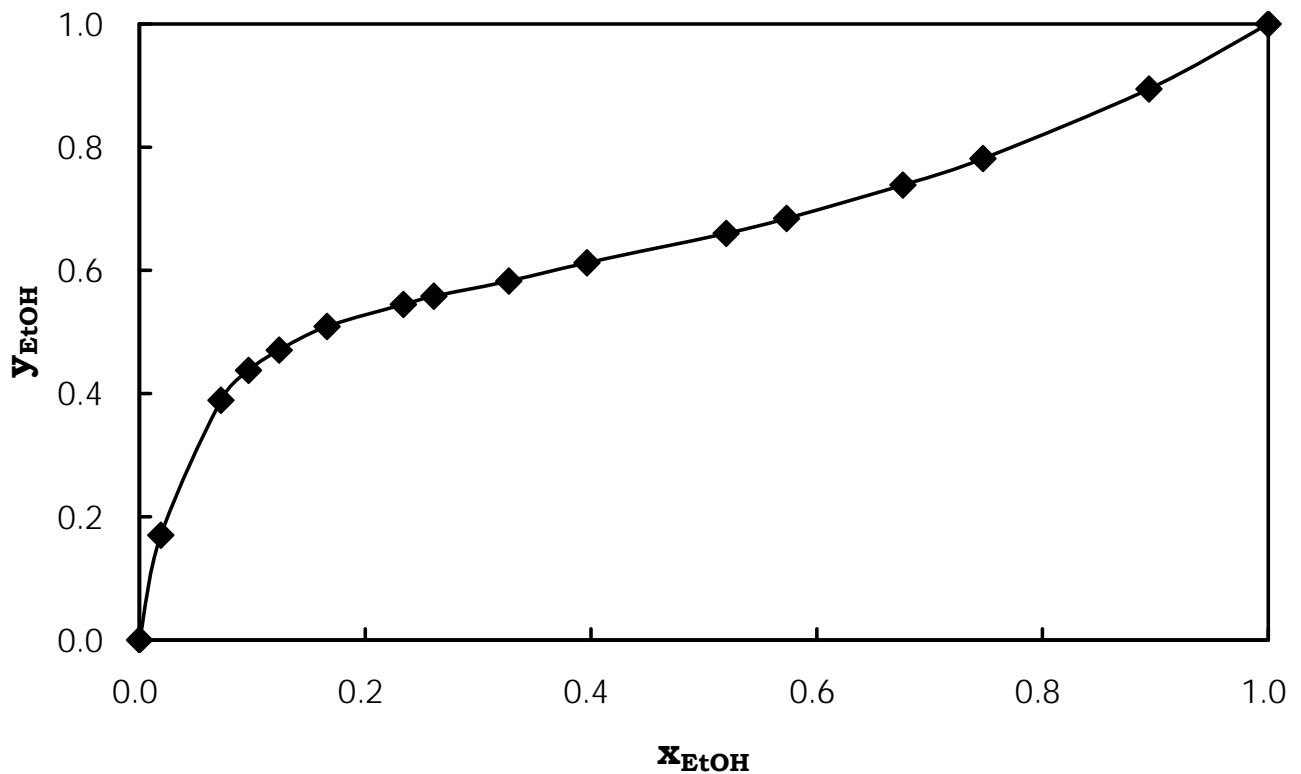
Hence,

$$\begin{aligned} \frac{\bar{L}}{\bar{V}} \text{ (for the stripping section)} &= \frac{740.0}{381.2} \\ &= 1.94 \end{aligned}$$

In order to perform the stage-by-stage calculations, the equilibrium data [between liquid phase ( $x$ ) and vapour phase ( $y$ )] is needed



Since the operating pressure is at 1 atm, we can use the following equilibrium data (curve) for the EtOH-water mixture (note that this is an  $x$ - $y$  plot for EtOH)



Let's start from the top of the column, where

$$y_1 = x_D = x_o = 0.61$$

Since the equilibrium is assumed, the mole fractions of the **liquid phase** ( $\mathbf{x}_n$ ) and the **vapour phase** ( $\mathbf{y}_n$ ) leaving any stage  $n$  are **in equilibrium** with each other; for example,  $x_1$  is in equilibrium with  $y_1$

Reading from the equilibrium curve, when  $y_1 = 0.61$ , we obtain

$$x_1 = 0.40$$

Then, the value of  $y_2$  can be calculated using Eq. 4.28:

$$y_{j+1} = \frac{L}{V} x_j + \left(1 - \frac{L}{V}\right) x_D \quad (4.28)$$

as follows

$$y_2 = \left[(0.75)(0.40)\right] + \left[(1 - 0.75)(0.61)\right] = 0.45$$

Once again, as  $x_2$  is in equilibrium with  $y_2$ , the value of  $x_2$  can be read from the equilibrium curve when  $y_2$  is known

From the equilibrium curve, when  $y_2 = 0.45$ , we obtain

$$x_2 = 0.11$$

Since stage 2 is the feed stage, to compute the value of  $y_3$ , the mole fraction of the vapour phase leaving stage 3, which is the stage below the feed stage (*i.e.* the *stripping* section), Eq. 4.29 (the operating equation for the *stripping* section):

$$y_k = \frac{\bar{L}}{\bar{V}} x_{k-1} - \left( \frac{\bar{L}}{\bar{V}} - 1 \right) x_B \quad (4.29)$$

must be used as follows

$$y_3 = \left[ (1.94)(0.11) \right] - \left[ (1.94 - 1)(0.02) \right]$$
$$y_3 = 0.195 \approx 0.20$$

Once again, from the equilibrium curve, when  $y_3 = 0.20$ , the value of the corresponding  $x_3$  is

$$x_3 = 0.02$$

Since  $x_3 = x_B = 0.02$ , **our calculations are now *finished***

This means that this column has **3 stages**

Actually, it has only **2 stages plus with a *re-boiler***, which can be considered as an *additional stage*

## 4.2 McCabe-Thiele Method

From the Lewis (CMO) technique illustrated in the recent Example, the *stage-by-stage* solution involves the **equilibrium curve** and the **operating lines**

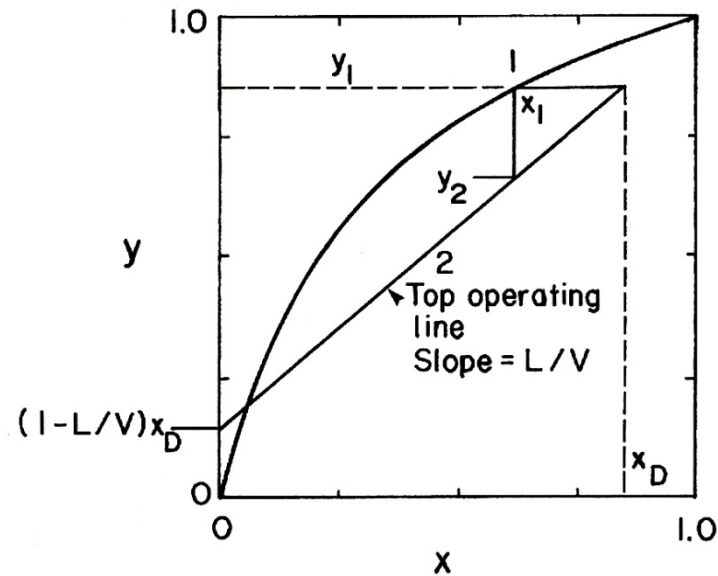
Note that there are **2 operating lines**:

- The operating line for *enriching* section (**Eq. 4.28**) – the ***TOP operating line***, as shown in Figure 4.7:

$$y_{j+1} = \frac{L}{V} x_j + \left(1 - \frac{L}{V}\right) x_D$$

in which  $\frac{L}{V} \equiv \text{slope}$  and  $\left(1 - \frac{L}{V}\right) x_D \equiv$

*Y-intercept*



**Figure 4.7: The *top* operating line**

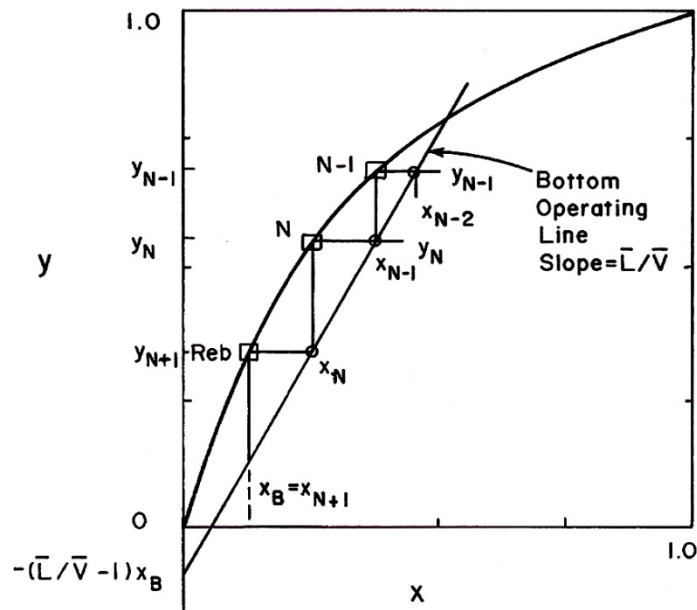
(from “Separation Process Engineering” by Wankat, 2007)

- The operating line for *stripping* section (Eq. 4.29) – the ***BOTTOM operating line***, as depicted in Figure 4.8:

$$y_k = \frac{\bar{L}}{\bar{V}} x_{k-1} - \left( \frac{\bar{L}}{\bar{V}} - 1 \right) x_B$$

in which  $\frac{\bar{L}}{\bar{V}} \equiv slope$  and  $-\left( \frac{\bar{L}}{\bar{V}} - 1 \right) x_B \equiv$

*Y-intercept*



**Figure 4.8: The *bottom* operating line**

(from “Separation Process Engineering” by Wankat, 2007)

W.L. McCabe and E.W. Thiele (1925) proposed a *graphical* technique to provide the solutions for the distillation problems as illustrated in **Figures 4.7 and 4.8**

Note that the *McCabe-Thiele* method is, theoretically, based on the technique developed by W.K. Lewis

To start **solving a distillation problem from the *top*** of the *column*, we follow the following **procedure**:

- 1)  $x_D$  must be specified, and since  $x_D = y_1$ , the value of  $y_1$  can then be specified
- 2) Once  $y_1$  is specified, the corresponding value of  $x_1$  can be read from the equilibrium curve
- 3) Since  $x_1$  and  $y_2$  are related to each other through Eq. 4.28 (the *operating line* for the *rectifying* section), the value of  $y_2$  can be obtained from the top operating line once  $x_1$  is specified
- 4) When  $y_2$  is obtained,  $x_2$  is, once again, can be obtained through the use of the equilibrium curve



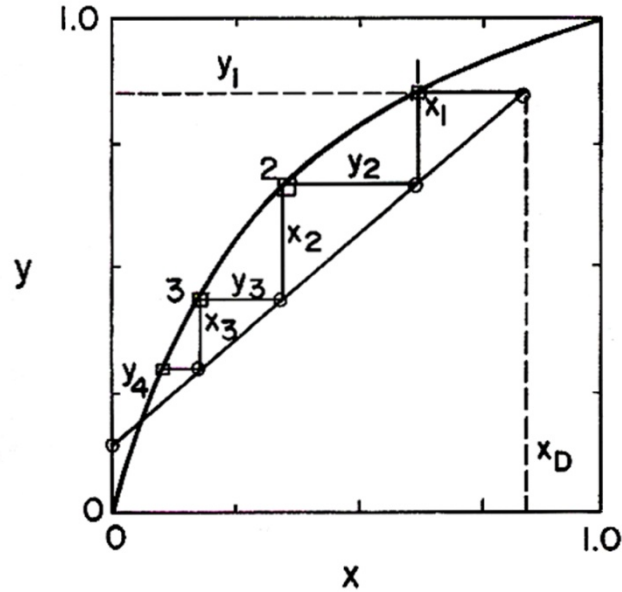
5) Repeat 1) – 4), as illustrated in Figure 4.9, until we reach the feed stage

After we enter the *stripping* section, the *bottom* operating line must be used (see Figure 4.8)

We, then, follow the same procedure until we obtain

$$x_{N+1} \leq x_B$$

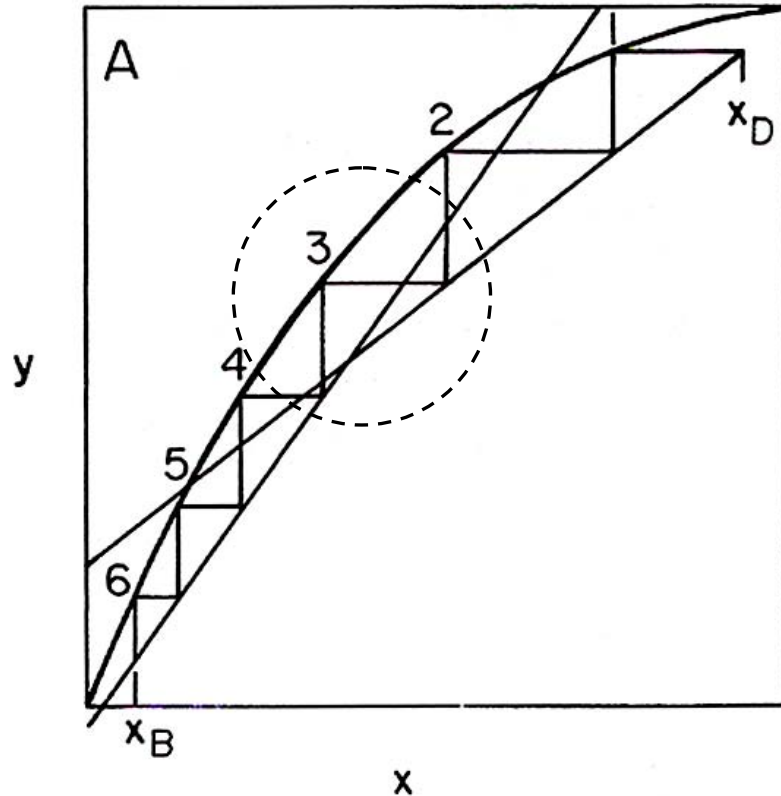
The McCabe-Thiele diagram for the whole column (*i.e.* both rectifying and stripping sections) can be illustrated in Figure 4.10



**Figure 4.9: An illustration of the use of McCabe-Thiele technique to solve the distillation problems for the rectifying/enriching section**

(from “Separation Process Engineering” by Wankat, 2007)

The number of steps or staircases (ขั้นบันได) from  $x_D$  to  $x_B$  is the number of stages of the distillation column



**Figure 4.10:** An illustration of the use of McCabe-Thiele technique to solve the distillation problems for the whole column (note that, in this Figure, the feed stage is stage 2 from the top)

(from “Separation Process Engineering” by Wankat, 2007)

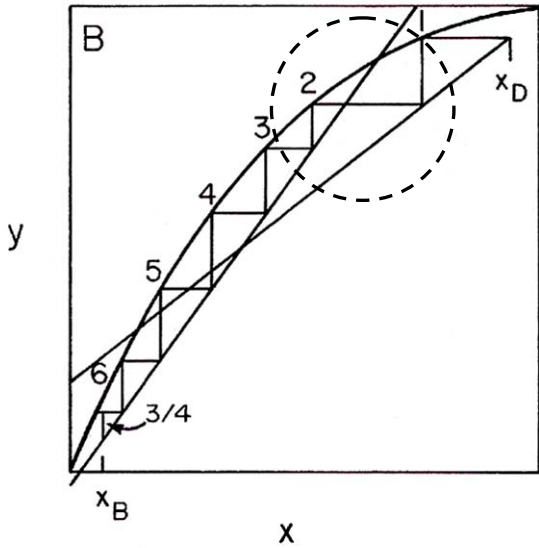
From Figure 4.10, we obtain the facts that, in this Example,

- the total number of stages = 6
- the feed stage = stage 3 (from the top)

Note that the *feed stage/tray* is the stage that **crosses** *from* the *rectifying* or *enriching section* *into* the *stripping* section, as depicted in Figure 4.10; note that, in Figure 4.10, it is the stage the **covers** the **intersection** of the *top* and the *bottom* operating lines

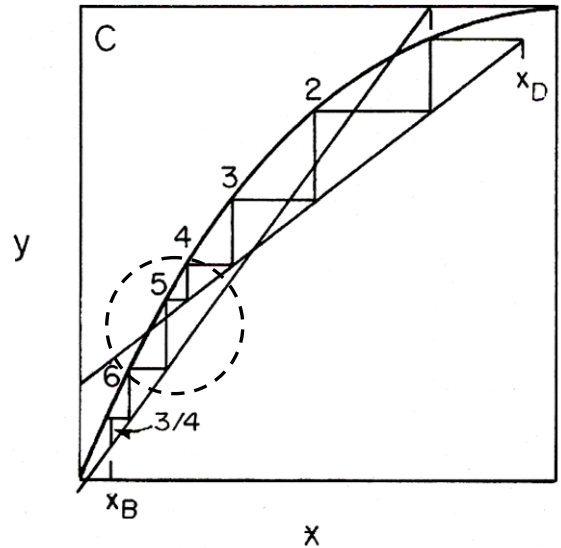
However, it is **NOT** always the case that the **feed stage** must be the **stage** that includes the *intersection* of the *top* and *bottom* operating lines, as shown in Figures 4.11 and 4.12

For the cases of Figures 4.11 and 4.12, the total number of stages are  $6\frac{3}{4} \approx 7$  stages, which are more than that for the case of Figure 4.10 (in which the total number of stages = 6)



**Figure 4.11**

(Feed stage = # 2)



**Figure 4.12**

(Feed stage = # 5)

(from “Separation Process Engineering” by Wankat, 2007)

For the *binary* mixtures, when the **feed stage** includes the **intersection of the top and bottom operating lines**, that feed stage is called the “*optimal feed stage*”, which yields the *minimum* number of stages of the distillation column

To draw the top and bottom operating lines, it requires  $\frac{L}{V}$  and  $\frac{\bar{L}}{\bar{V}}$ , respectively, as the slopes

Practically, however, it is difficult to obtain the values of  $\frac{L}{V}$  and  $\frac{\bar{L}}{\bar{V}}$ , as these are the *internal* flow rates (within the distillation column)

It would be much *more convenient* or *more practical* if we can obtain the **values of**  $\frac{L}{V}$  and  $\frac{\bar{L}}{\bar{V}}$  from the *external flow rates*, which can be derived as follows

Since

$$V = L + D \quad (4.44)$$

Thus,

$$\frac{L}{V} = \frac{L}{L + D}$$

which can be re-arranged to

$$\frac{L}{V} = \frac{\frac{L}{D}}{\frac{L}{D} + 1} \quad (4.45)$$

where  $\frac{L}{D} = \mathbf{reflux\ ratio}$

For the value of  $\frac{\bar{L}}{\bar{V}}$ , which is used for drawing the *bottom* operating line, since the **boil-up ratio**  $\left(\frac{\bar{V}}{B}\right)$  is practically NOT specified, we cannot calculate the value of  $\frac{\bar{L}}{\bar{V}}$  in the same manner that we do for  $\frac{L}{V}$

In order to obtain the value of  $\frac{\bar{L}}{\bar{V}}$ , the *feed*

*quality*:

$$q = \frac{\bar{L} - L}{F} \approx \frac{H - h_F}{H - h} \quad (4.36b)$$

is employed

Eq. 4.36b can be re-arranged to

$$\bar{L} - L = qF \quad (4.37a)$$

and

$$\bar{L} = L + qF \quad (4.37b)$$

The overall material balance at the re-boiler  
(see Page 12) is

$$\bar{L} = B + \bar{V} \quad (4.46)$$

which can be re-arranged to

$$\bar{V} = \bar{L} - B \quad (4.47)$$



By re-arranging the *external* overall and species balance equations (in Chapter 3):

$$F = B + D \quad (3.4)$$

and

$$z_F F = Bx_B + Dx_D \quad (3.5)$$

we obtain the following equations

$$D = \left( \frac{z_F - x_B}{x_D - x_B} \right) F \quad (4.48)$$

and

$$B = F - D = \left( \frac{x_D - z_F}{x_D - x_B} \right) F \quad (4.49)$$

Combining Eqs. 4.37b and 4.47 with Eqs. 4.48 and 4.49 and re-arranging the resulting equation for  $\frac{\bar{L}}{\bar{V}}$  results in

$$\frac{\bar{L}}{\bar{V}} = \frac{\frac{L}{D}(z_F - x_B) + q(x_D - x_B)}{\frac{L}{D}(z_F - x_B) + q(x_D - x_B) - (x_D - z_F)} \quad (4.50)$$

### 4.3 Feed Line

When it is assumed that the **CMO** (which is proposed by W.K. Lewis) is *valid*, the species balance (for a *more volatile* component) equations for the *enriching* (e.g., Eq. 4.2, 4.6, or 4.10) and the *stripping* (e.g., Eq. 4.14 or 4.18) sections can, respectively, be formulated as follows

$$yV = xL + x_D D \quad (4.51)$$

and

$$y\bar{V} = x\bar{L} - x_B B \quad (4.52)$$

Since Eqs. 4.51 and 4.52 represent the TOP and BOTTOM operating lines, respectively, the intersection of these two lines means

$$y_{\text{top op}} = y_{\text{bot op}} \quad (4.53)$$

and

$$x_{\text{top op}} = x_{\text{bot op}} \quad (4.54)$$

As we have learned previously, the *optimal feed point* is the feed stage that covers the *intersection of the two operating lines*; thus, to solve for the value of  $y_i$  at the point where  $y_{\text{top op}} = y_{\text{bot op}}$ , we do the following

(4.52) – (4.51) results in

$$y(\bar{V} - V) = (\bar{L} - L)x - (x_D D + x_B B) \quad (4.55)$$

Substituting the *external* balance equation for a *more volatile* species or Eq. 3.5:

$$z_F F = Bx_B + Dx_D \quad (3.5)$$

into Eq. 4.55 yields

$$y(\bar{V} - V) = (\bar{L} - L)x - z_F F \quad (4.56)$$

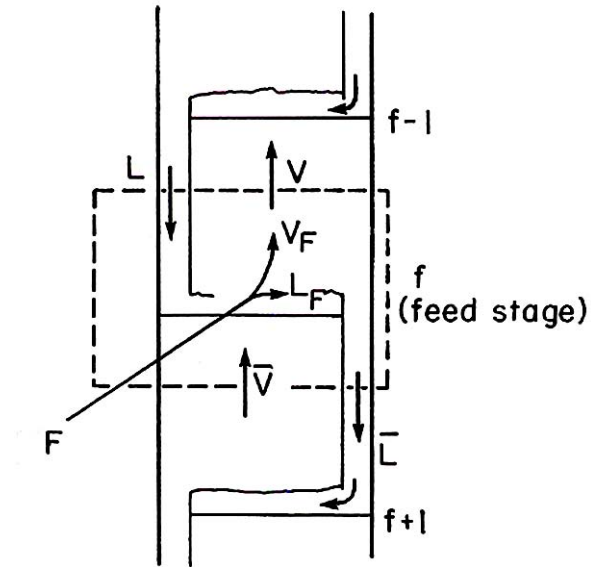
Re-arranging Eq. 4.56 gives

$$y = \frac{(\bar{L} - L)}{(\bar{V} - V)}x - \frac{z_F F}{(\bar{V} - V)} \quad (4.57a)$$

or

$$y = -\frac{(\bar{L} - L)}{(V - \bar{V})}x + \frac{z_F F}{(V - \bar{V})} \quad (4.57b)$$

Consider the feed stage as shown in Figure 4.13, in which the feed ( $F$ ) is divided into the liquid phase ( $L_F$ ) and the vapour phase ( $V_F$ )



**Figure 4.13: A feed stage**

(from “Separation Process Engineering” by Wankat, 2007)

Performing material (either mole or mass) balances at the feed stage for the liquid and the vapour phases gives

$$L + L_F = \bar{L} \quad (4.58a)$$

or

$$\bar{L} - L = L_F \quad (4.58b)$$

and

$$\bar{V} + V_F = V \quad (4.59a)$$

or

$$V - \bar{V} = V_F \quad (4.59b)$$

Substituting Eqs. 4.58b and 4.59b into Eq. 4.57b results in

$$y = -\frac{L_F}{V_F} x + \frac{F}{V_F} z_F \quad (4.60)$$

Dividing the numerator (๒๗๒) and the denominator (๓๖๗) of the terms on the RHS of Eq. 4.60 by  $F$  yields

$$y = -\frac{L_F / F}{V_F / F} x + \frac{F / F}{V_F / F} z_F \quad (4.61)$$

When combining Eq. 4.61 with the following definitions previously defined in Chapter 2:

$$\frac{V_F}{F} = f \quad (4.62)$$

$$\frac{L_F}{F} = 1 - f \quad (4.63)$$

we obtain

$$y = -\frac{1-f}{f}x + \frac{1}{f}z_F \quad (4.64)$$

Alternatively, as

$$F = L_F + V_F \quad (4.65)$$

by dividing Eq. 4.65 with  $F$  we obtain

$$\frac{F}{F} = \frac{L_F}{F} + \frac{V_F}{F}$$

$$\frac{L_F}{F} + \frac{V_F}{F} = 1 \quad (4.66a)$$

or

$$q + f = 1 \quad (4.66b)$$

Accordingly,

$$q = 1 - f = 1 - \frac{V_F}{F} \quad (4.67)$$

or

$$\frac{V_F}{F} = 1 - q = 1 - \frac{L_F}{F} \quad (4.68)$$

Thus, Eq. 4.61 can be written in another form as follows

$$y = -\frac{L_F / F}{V_F / F} x + \frac{F / F}{V_F / F} z_F$$
$$y = -\frac{q}{1 - q} x + \frac{1}{1 - q} z_F \quad (4.69)$$

In addition to the fact that Eq. 4.69 is equivalent to Eqs. 4.61 and 4.64, Eqs. 4.64, and 4.69 are, in fact, Eq. 2.8:



$$y_i = -\frac{1-f}{f}x_i + \frac{z_i}{f} \quad (2.8)$$

and Eq. 2.11:

$$y_i = -\frac{q}{1-q}x_i + \left(\frac{1}{1-q}\right)z_i \quad (2.11)$$

in Chapter 2, respectively

Eq. 4.69:

$$y = -\frac{q}{1-q}x + \frac{1}{1-q}z_F \quad (4.69)$$

is commonly called the “feed line” equation

(although *Eq. 4.64* is *equivalent* to this equation, the *feed-line* equation is commonly *written* in the form of **Eq. 4.69**)

Let's consider Eq. 4.36b:

$$q = \frac{\bar{L} - L}{F} \approx \frac{H - h_F}{H - h} \quad (4.36b)$$

once again

When the **feed** enters the distillation column as a *saturated liquid*; *i.e.*

$$F = L_F = \bar{L} - L \quad (4.70)$$

(see Eq. 4.58b on Page 53)

it results in the fact that

$$q = \frac{\bar{L} - L}{F} = \frac{F}{F} = 1$$

With  $q = 1$ , the slope of Eq. 4.69 becomes

$$-\frac{q}{1-q} = \frac{q}{q-1} = \frac{(1)}{(1)-1} = \frac{1}{0} = \infty$$

which means, mathematically, that the **feed line** is a *vertical* line (เส้นแนวตั้ง)

In the case that the **feed** enters the column as a *saturated vapour*; it results in the fact that  $q = 0$  as there is no liquid in the feed

With  $q = 0$ , the slope of Eq. 4.69 becomes

$$-\frac{q}{1-q} = \frac{q}{q-1} = \frac{(0)}{(0)-1} = 0$$

which implies, mathematically, that the **feed line** is a *horizontal* line (เส้นแนวนอน)

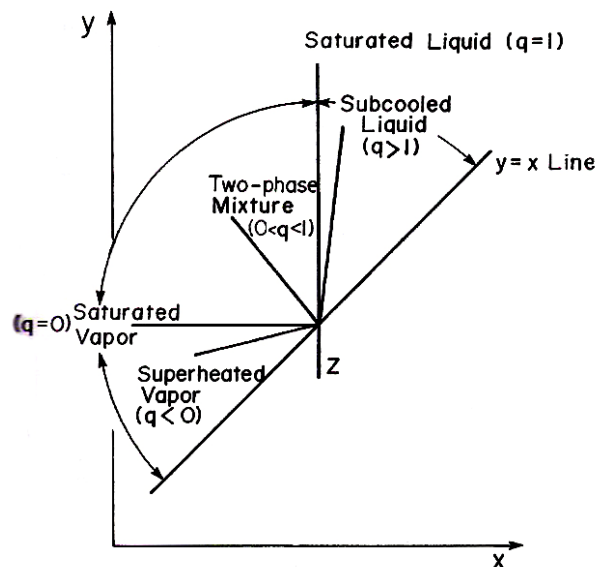
Note that, as we have already learned from Chapter 2, the **feed line** (either it is a *vertical* or *horizontal* line) intersects with the  $y = x$  line at

$$y = x = z_F$$

The *feed line* for other conditions of the feed, which include

- a saturated *liquid-vapour* mixture  
( $0 < q < 1$ )
- a superheated vapour ( $q < 0$ )
- a compressed or sub-cooled liquid ( $q > 1$ )

are shown, along with the case of a saturated liquid and a saturated vapour, in Figure 4.14



**Figure 4.14: The feed line for various conditions of the feed**

(from “Separation Process Engineering” by Wankat, 2007)

Note that for a **superheated vapour** and a **compressed liquid**, the *slopes* are *positive*

There are two major ways to operate the distillation column:

1) The feed condition is fixed, while the re-

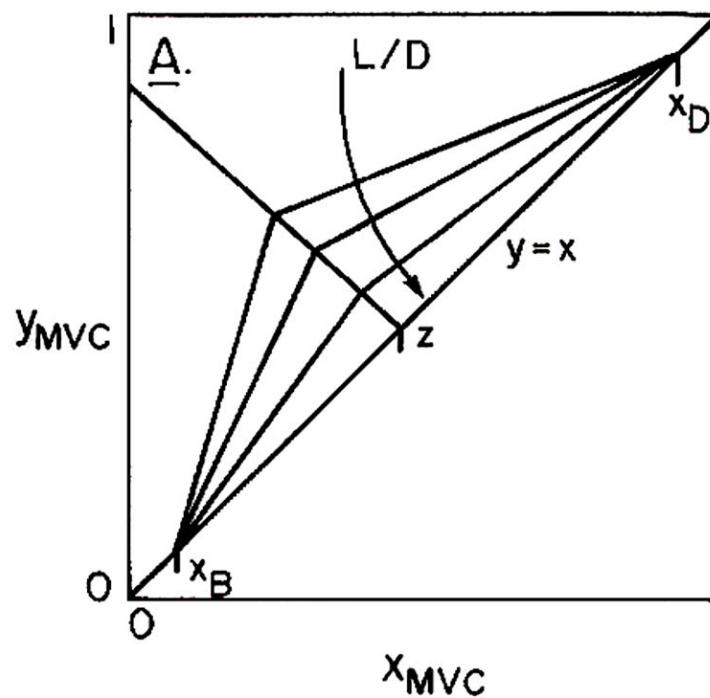
flux ratio  $\left(\frac{L_o}{D}\right)$  is varied

2) The reflux ratio is fixed, while the feed condition is varied

For the first case, when the **reflux ratio**  $\left(\frac{L_o}{D}\right)$  is **varied**,  $L$  and  $\bar{L}$  are also varied, while the feed is fixed, which means that the feed line is fixed, it results in the **changes in the slopes of**

- the *top* operating line
- the *bottom* operating line

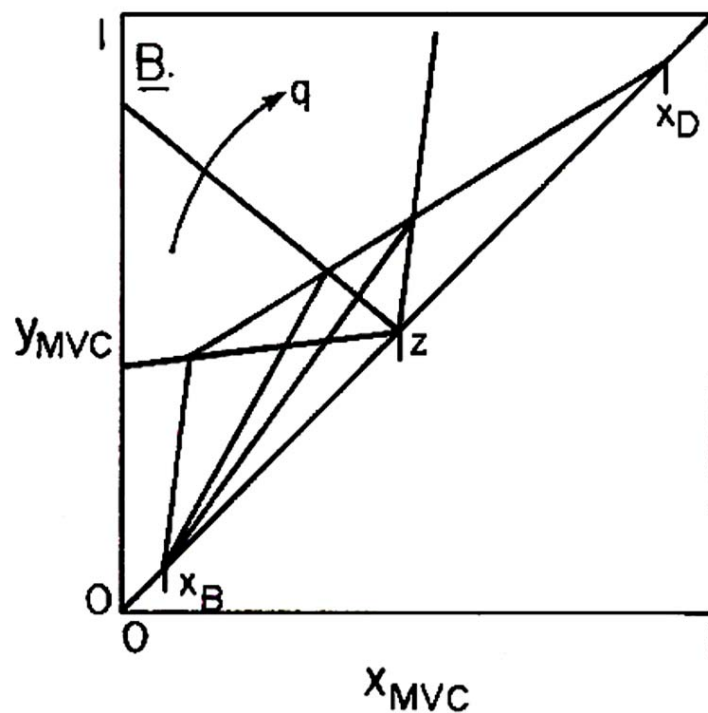
during the operation of the distillation column as illustrated in Figure 4.15



**Figure 4.15:** The top & bottom operating lines and the feed line for the operation that the feed condition is fixed, while the reflux ratio is varied

(from “Separation Process Engineering” by Wankat, 2007)

For the second case, when the distillation column is operated in the manner that the reflux ratio is fixed, while the operating condition is varied, the top operating line is constant, while the feed line and the bottom operating line are varied, as illustrated in Figure 4.16



**Figure 4.16: The top & bottom operating lines and the feed line for the operation that the reflux ratio is fixed, while the feed condition is varied**

(from “Separation Process Engineering” by Wankat, 2007)

In either operating condition, it should be emphasised once again that the *optimal* feed stage (which yields the minimum number of stages) is the stage in covers the intersection of the top and bottom operating lines

To determine the compositions (normally, of the *more volatile* species) in the liquid and the vapour phases, we can do as follows

Let the co-ordinate of the intersection of the top and bottom operating lines be  $(x_I, y_I)$

Hence, the *optimal* feed stage is the stage that makes

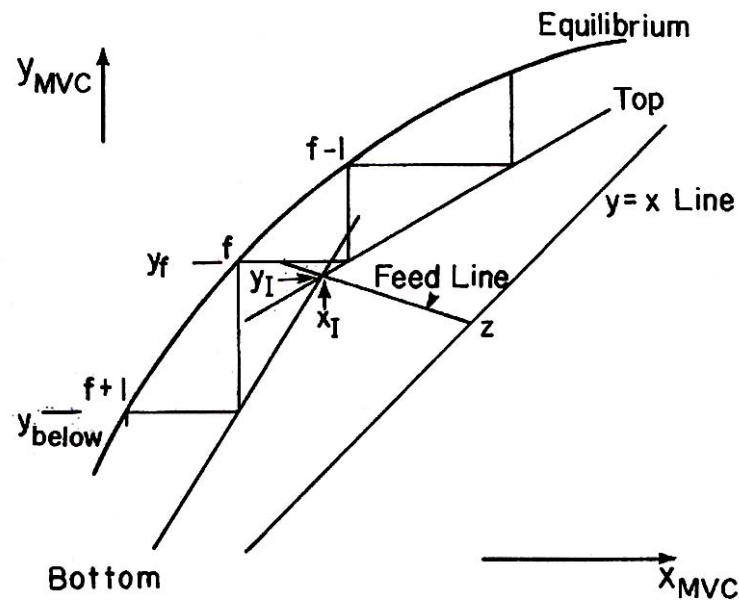


$$y_{f-1} < y_I < y_f$$

and

$$x_f < x_I < x_{f-1}$$

as illustrated in Figure 4.17



**Figure 4.17: The feed line for the optimal feed stage** (note that *MVC* stands for *more volatile component*)

(from “Separation Process Engineering” by Wankat, 2007)

By employing the facts that, at the intersection of the top and bottom operating lines,

$$y_{\text{top op}} = y_{\text{bot op}} \quad (4.53)$$

and

$$x_{\text{top op}} = x_{\text{bot op}} \quad (4.54)$$

the values of  $x_I$  and  $y_I$  can be solved mathematically, and the solutions are

$$x_I = \frac{-(q-1)\left(1 - \frac{L}{V}\right)x_D - z_F}{(q-1)\left(1 - \frac{L}{V}\right) - q} \quad (4.71)$$

and

$$y_I = \frac{z_F + \frac{qx_D}{(L/D)}}{1 + \frac{q}{(L/D)}} \quad (4.72)$$

**Example** Calculate the slope  $\left( -\frac{q}{1-q} = \frac{q}{q-1} \right)$

of the feed lines for the following cases

- a) A *two-phase* feed where 80% of the feed is *vaporised* when it enters the column
- b) A *superheated vapour* feed where 1 mole of liquid will vaporise at the feed stage for each of every 9 moles of the feed
- c) A *sub-cooled liquid* feed that is fed to the column at 35 °F below its boiling-point temperature

Given that the specific heat ( $c_p$ ) of the feed ( $F$ ) is 30 Btu/(lb-mol-°F) and latent heat of vaporisation/condensation ( $\lambda$ ) of the feed is 15,000 Btu/lb-mol

a)

It is given that 80% of the feed is *vaporised* when it enters the column, which can be written as follows

$$V_F = 0.8F$$

Thus, it results in the fact that

$$L_F = 0.2F$$

Therefore, the value of  $q$  can be computed using Eqs. 4.36b and 4.70 (see Page 58) as follows

$$q = \frac{\bar{L} - L}{F} = \frac{L_F}{F} = \frac{0.2F}{F}$$

$$q = 0.2$$

The *slope* of the *feed line* can, then, be calculated as follows

$$\text{Slope of the feed line} = \frac{q}{q - 1} = \frac{0.2}{0.2 - 1} = -0.25$$

This complies (agrees) with Figure 4.14 (for the *two-phase mixture* case), as the *slope* is *negative*

**b)**

Since the feed is a “*superheated vapour*” (*i.e.* it is hotter than a “*saturated vapour*”), some amount of liquid that *flows down* from *rectifying* section (with the flow rate of  $L$ ) is boiled and vaporised, and the *amount of liquid vaporised* is

$$L - \bar{L} = (\text{amount of liquid vaporised: } l_v) \quad (4.73a)$$

or

$$\bar{L} - L = -(\text{amount of liquid vaporised, } l_v) \quad (4.73b)$$

It is given, in the problem statement, that **liquid** in the amount of **1 mol** is *vaporised per 9 mol* of the **feed**

Hence, the *amount of liquid* that is *vaporised* can be written mathematically as

$$(\text{amount of liquid vaporised, } l_v) = \frac{1}{9} F$$

Accordingly,

$$\bar{L} - L = -\frac{1}{9} F$$

and, when combined with Eq. 4.36b, we obtain the value of  $q$  as follows

$$q = \frac{\bar{L} - L}{F} = \frac{-\frac{1}{9}F}{F} = -\frac{1}{9}$$

Eventually, the *slope* of the *feed line* for this feed condition (a *superheated vapour*) can be computed as follows

$$\text{Slope} = \frac{q}{q - 1} = \frac{\left(-\frac{1}{9}\right)}{\left(-\frac{1}{9}\right) - 1} = \frac{1}{10}$$

Note that the *slope* is *positive*, which agrees with Figure 4.14 (when the feed is a *superheated vapour*)

c)

In this case, the feed is a “*sub-cooled*” liquid (*i.e.* it is cooler than a “saturated liquid”), when it enters the column, it causes a condensation of the vapour that flows up from the *stripping* section (with the flow rate of  $\bar{V}$ )

The amount of vapour that is condensed when it is cooled by the *sub-cooled liquid* feed is

$$\bar{V} - V = (\text{amount of vapour condensed: } c)$$

Thus, in addition to the *compressed* (or *sub-cooled*) *liquid* feed, there is also the liquid that comes from the condensation of the vapour that flows up from the stripping section



Hence, in total, the liquid flow rate below the feed stage ( $\bar{L}$ ) is

$$\bar{L} = L + F + (\text{amount of vapour condensed: } c) \quad (4.73)$$

Since, normally, the *distillation column* is assumed to be *adiabatic* (see the assumptions for the CMO technique on Pages 13-14) or there is no heat moving in and out of the column, the heat generated from the condensation of the vapour phase is used to heat up the feed (which is a sub-cooled liquid)

The amount of heat generated from the condensation of the vapour phase ( $Q_{\text{condensation}}$ ) is

$$Q_{\text{condensation}} = c\lambda \quad (4.74)$$

where  $\lambda$  is the latent heat of vaporisation/condensation

In parallel, the amount of heat required to heat up the *sub-cooled liquid* feed (to its boiling point) ( $Q_{\text{heat up}}$ ) is

$$Q_{\text{heat up}} = Fc_p \Delta T \quad (4.75)$$

where

$F$  = the feed flow rate

$c_p$  = a specific heat of the feed

$\Delta T$  = a difference between the boiling temperature of the feed and its actual (feeding) temperature

By employing the *assumption* that the *distillation* column is *adiabatic*, we obtain the fact that

$$\begin{aligned} Q_{\text{condensation}} &= Q_{\text{heat up}} \\ c\lambda &= Fc_p \Delta T \end{aligned} \quad (4.76)$$

Substituting corresponding numerical values into Eq. 4.76 and solving for  $c$  yields

$$\begin{aligned} F(30)(35) &= c(15,000) \\ c &= \frac{(30)(35)}{(15,000)} F = 0.07F \end{aligned}$$

Thus, in this Example, by using Eq. 4.73, the flow rate of the liquid phase below the feed stage is

$$\bar{L} = L + F + 0.07F$$

which can be re-arranged to

$$\bar{L} - L = F + 0.07F = 1.07F$$

Accordingly, the value of  $q$  can be computed using Eq. 4.36b as follows

$$q = \frac{\bar{L} - L}{F} = \frac{1.07F}{F} = 1.07$$

With the value of  $q$  of 1.07, the *slope* of the *feed line* can then be calculated as follows

$$\text{Slope} = \frac{q}{q - 1} = \frac{(1.07)}{(1.07) - 1} = 15.3$$

Note, once again, that the slope is positive, which agrees with Figure 4.14 (for the case that the feed is a *sub-cooled liquid*)

An illustration of the use of the *graphical McCabe-Thiele method* is exemplified in the following Example

**Example** A distillation column, at 1 atm, with a *total* condenser and a *partial* re-boiler is used to separate an ethanol (EtOH)-water mixture

The feed is 20 mol% EtOH with a feed rate of 1,000 kmol/h ( $h_F = 25$  Btu/lb)

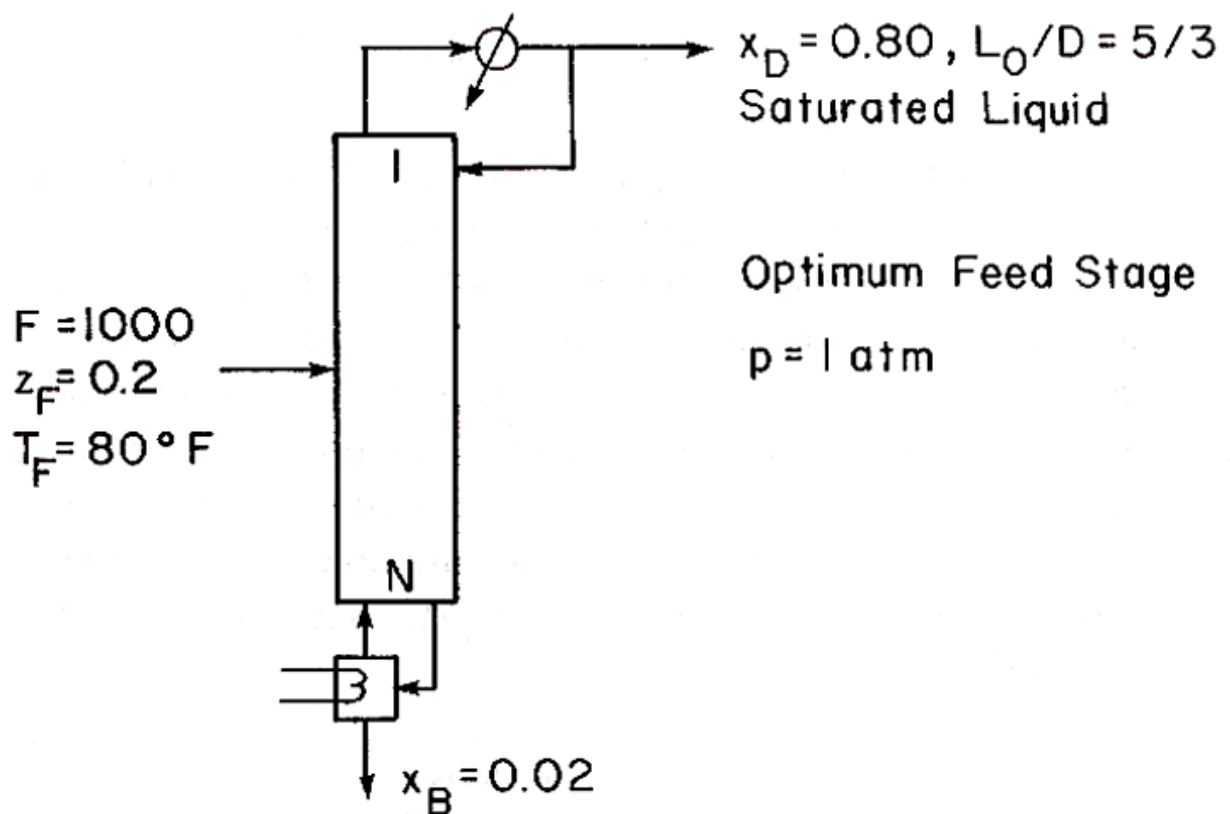
A *distillate* composition of 80 mol% EtOH and a *bottom* composition of 2 mol% EtOH are required

The external *reflux ratio* is 5/3, and the *reflux* is a *saturated liquid*

If the *CMO* is *assumed*, find the *number of equilibrium stages* and the *optimal feed stage*

Given the enthalpies for a saturated vapour ( $H$ ) and a saturated liquid ( $h$ ) of 880 and 125 Btu/lb, respectively

The problem statement can be described as the following flow chart:



(from "Separation Process Engineering" by Wankat, 2007)

To solve this distillation problem, the following *standard procedure* is to be carried out

- 1) Find *equilibrium data* for a given mixture and pressure and then draw a  $y-x$  diagram (normally for a *more volatile* component: MVC) – *e.g.*, the  $y-x$  diagram of EtOH on Page 33
- 2) Check if the *CMO* is *valid*

The easiest way is to *check* the *latent heat of vaporisation* of each component and *determine* that whether or not *their values* are *close to each other* (normally  $\pm 10\%$  difference is acceptable)

In this Example,  $\lambda_{\text{EtOH}} = 9.2 \text{ kcal/mol}$  and  $\lambda_{\text{water}} = 9.7 \text{ kcal/mol}$  – close enough ( $\pm \sim 5\text{-}6\%$ ) to assume that CMO is valid

- 3) Locate the  $y_1 = x_D$  point (using the  $y = x$  line)
- 4) Calculate the value of  $\frac{L}{V}$  (for the *enriching/rectifying* section)
- 5) Draw the **top operating line** (in which the values of  $\frac{L}{V}$  or the Y-intercept of the top operating line and  $x_D$  are required)
- 6) Calculate/determine the value of  $q$
- 7) Draw the **feed line** (note that, in some textbooks, this line is called the “**q**” line)
- 8) Draw the **bottom operating line**
- 9) Step off the stages [either bottom up—*i.e.* from the re-boiler ( $x_B$ ) or top down—*i.e.* from the condenser ( $x_D$ )]



It is given, in the problem statement, that

$$x_D = 0.80 \text{ (80 mol\% EtOH)}$$

Thus, the point  $y_1 = x_D = 0.80$  can be located using the  $y = x$  line (this  $y_1 = x_D = 0.80$  point is on the  $y = x$  line)

The value of  $\frac{L}{V}$  can be computed using Eq.

4.45, as the value of reflux ratio  $\left(\frac{L_o}{D} \text{ or } \frac{L}{D}\right)$  is

given, as follows

$$\frac{L}{V} = \frac{\frac{L}{D}}{\frac{L}{D} + 1} = \frac{\frac{5}{3}}{\frac{5}{3} + 1} = \frac{5}{8}$$

As  $\frac{L}{V}$  is the **slope** of the *top operating line*,  
with this slope  $\left(\frac{L}{V}\right)$  and the **value** of  $x_D$ , the *top operating line* can be **drawn**

Alternatively, the **Y-intercept** of the *top operating line*:

$$y = \frac{L}{V}x + \left(1 - \frac{L}{V}\right)x_D \quad (4.28)$$

is used **along with** the **value** of  $x_D$  to **draw** the *top operating line*

In this Example,

$$\text{Y-intercept} = \left(1 - \frac{L}{V}\right)x_D = \left(1 - \frac{5}{8}\right)(0.80) = 0.30$$

From this Y-intercept ( $y = 0.30$ ) along with the point on the  $y = x$  line where  $y_1 = x_D = 0.80$ , the top operating line can also be drawn

To calculate the value of  $q$  for this Example, Eq. 4.36b is used as follows

$$q = \frac{H - h_F}{H - h} = \frac{880 - 25}{880 - 125} = 1.13$$

Thus, the slope of the feed line  $\left( i.e. \frac{q}{q-1} \right)$

can be calculated as follows

$$\text{Slope} = -\frac{q}{1-q} = \frac{q}{q-1} = \frac{1.13}{1.13-1} = 8.7$$

To draw the feed line, in addition to its *slope*, the *origin point* of the *feed line* is also required

As we have learned previously, the *origin point* of the *feed line* is *on* the  $y = x$  *line* at the point where

$$y = x = z_F$$

In this Example, the origin point of the feed line is at the point where

$$y = x = z_F = 0.20$$

If the feeding point is the *optimal* feed stage, it must include the intersection of the top and bottom operating lines

By employing this principle, the *feed line* should **pass through** the **intersection** of the *top* and *bottom* operating lines

Accordingly, one point of the bottom operating line is now located (or fixed) – *i.e.* at the intersection of the top and the feed lines

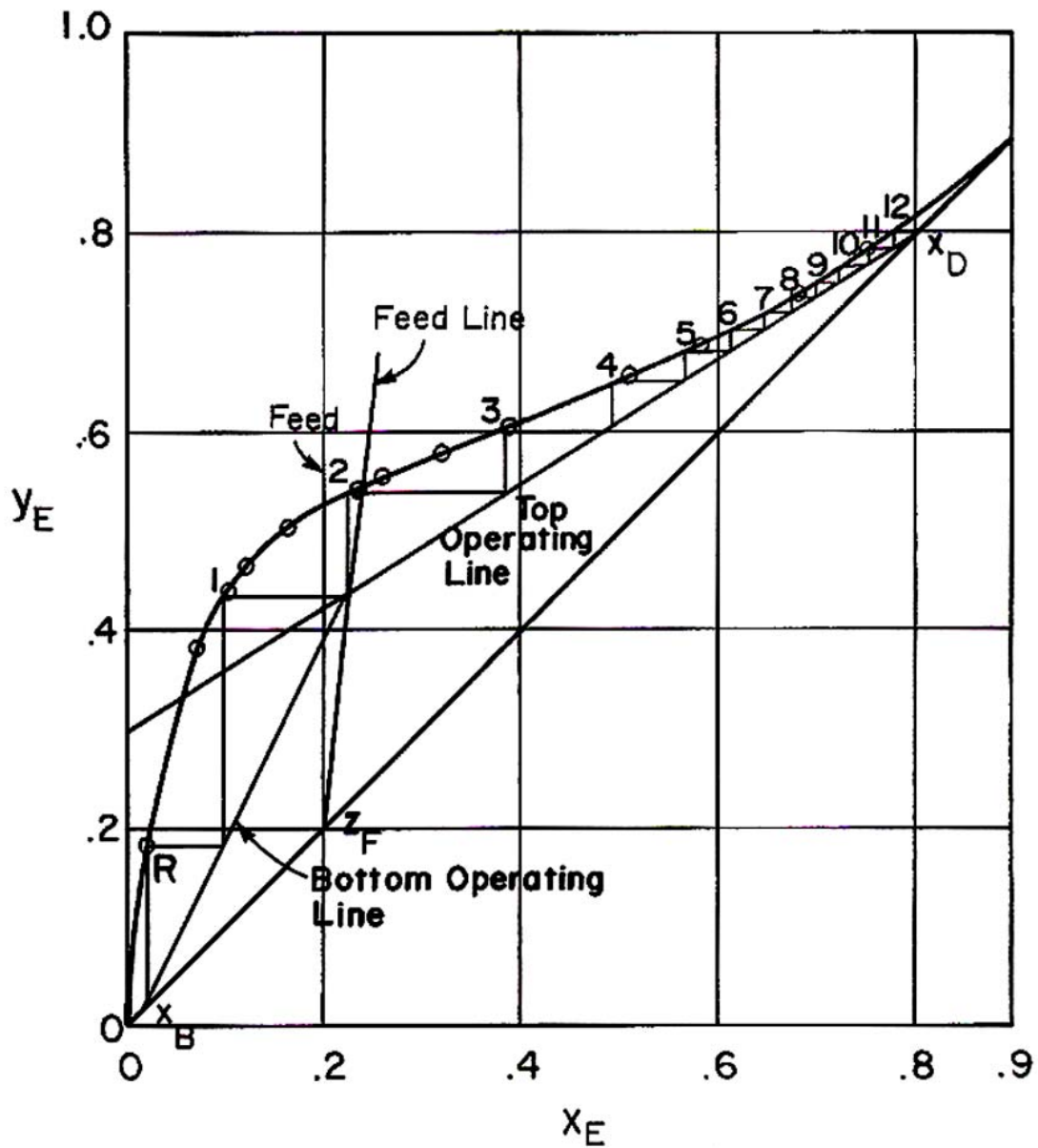
Another point through which the bottom operating line is passed is on the  $y = x$  line at the point where  $y = x = x_B$

In this Example,  $x_B = 0.02$  (2 mol% EtOH)

Then, as we put together:

- the equilibrium curve
- the top operating line
- the feed line
- the bottom operating line

on the same co-ordinate (*i.e.* the McCabe-Thiele diagram), as illustrated in Figure 4.18



**Figure 4.18:** An illustration of use of the graphical McCabe-Thiele technique to obtain the solutions for the distillation problem

(from “Separation Process Engineering” by Wankat, 2007)

we obtain the following answers:

- the number of *equilibrium* stages required
- the *optimal* feed stage/location

In **this Example**,

- the **total number** of *equilibrium* stages  
= **12 + a re-boiler = 13 stages**
- the *optimal* feed stage/location = **stage  
1 or 2 (from the re-boiler)**

Note that, *in this Example*, as the *feed* is a *sub-cooled liquid* (do you know why?)

## 4.4 Profiles for Binary Distillation

From Figure 4.18, the values of  $x_{\text{EtOH}}$  and  $y_{\text{EtOH}}$  at each stage can be read from the graph; for example, the values of  $x_{\text{EtOH}}$  and  $y_{\text{EtOH}}$  for the stream leaving the re-boiler are 0.02 and 0.18, respectively

Since

$$x_{\text{water}} = 1 - x_{\text{EtOH}}$$

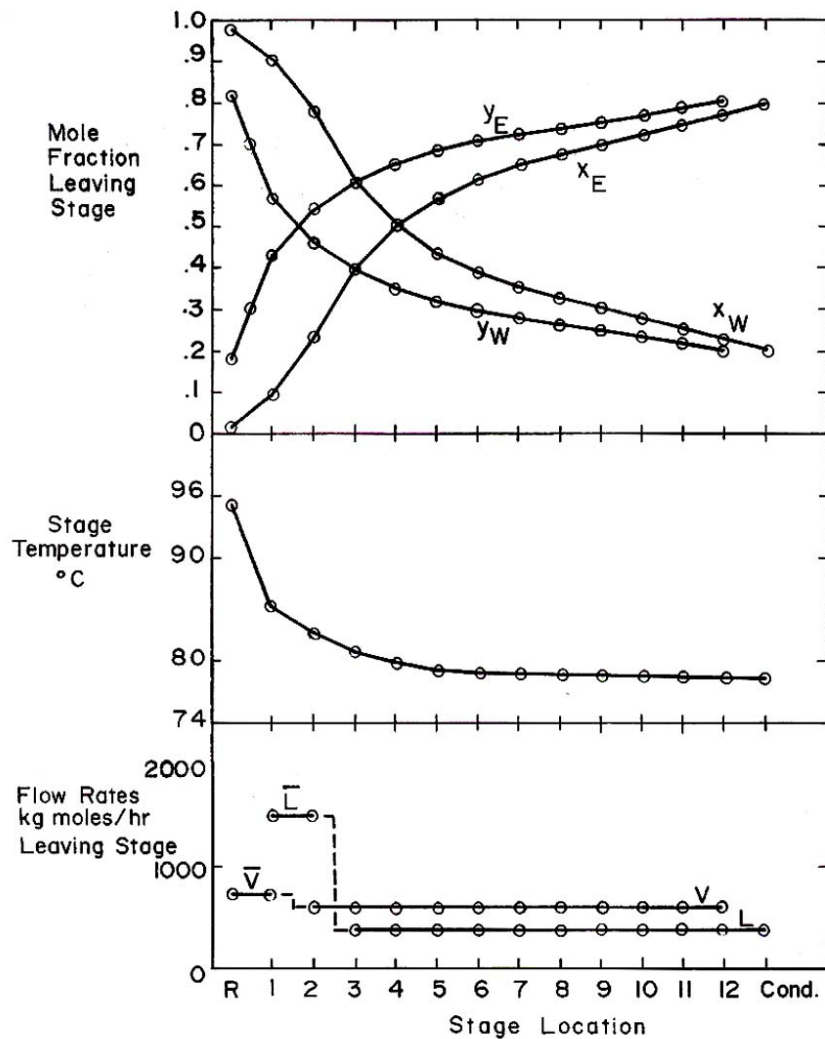
and

$$y_{\text{water}} = 1 - y_{\text{EtOH}}$$

the value of  $x_{\text{water}}$  and  $y_{\text{water}}$  of each stage can also be obtained



Therefore, the *concentration profiles* for the *vapour phase* ( $y_i$  of both EtOH and water) and the *liquid phase* ( $x_i$  of both EtOH and water) can be plotted and depicted in Figure 4.19



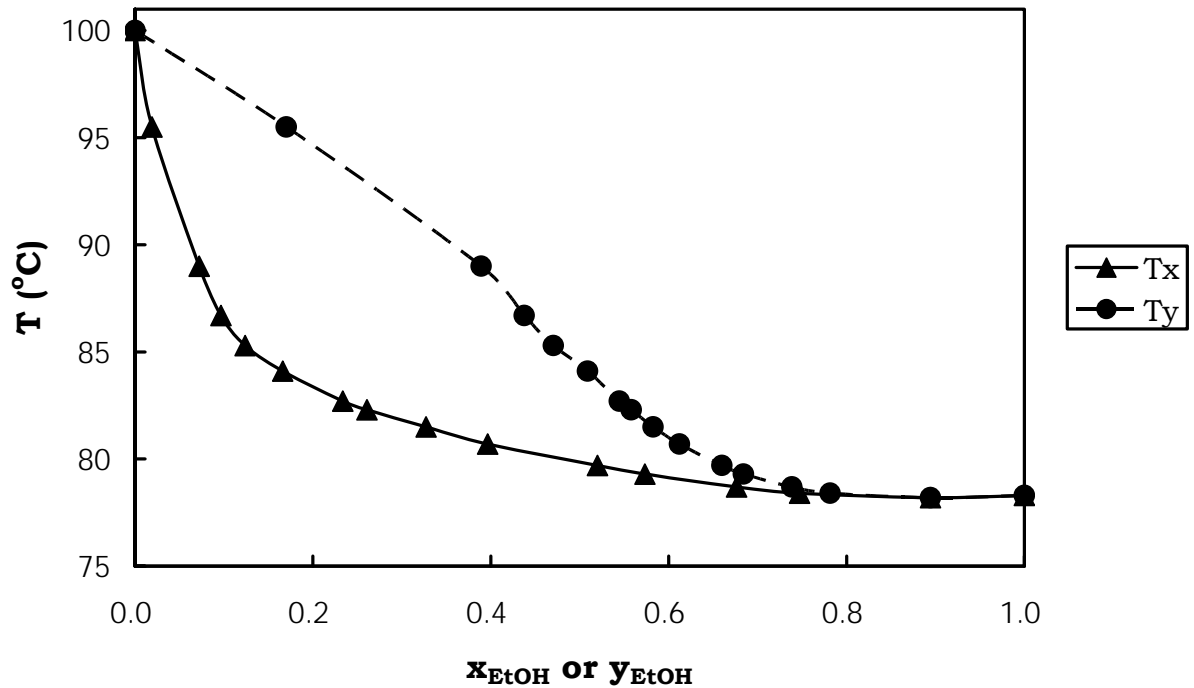
**Figure 4.19: The concentration, temperature, and flow rate profiles of the distillation column**

(from “Separation Process Engineering” by Wankat, 2007)

The *temperature profile* of the distillation column, also shown in Figure 4.19, is obtained when the values of  $x_i$  and  $y_i$  of each species for each stage are known

Once the values of  $x_i$  and  $y_i$  can be specified, we can use a ***Txy* diagram** (*e.g.*, Figure 4.20) to **determine the temperature of each stage**; for Example, for  $x_{\text{EtOH}} = 0.02$  and  $y_{\text{EtOH}} = 0.18$ , the temperature can be read, from Figure 4.20, as  $\sim 95.5\text{ }^\circ\text{C}$

The *flow rate profile* for the rectifying and stripping sections can also be plotted and included in Figure 4.19



**Figure 4.20: A  $Txy$  diagram for ethanol (EtOH) for an EtOH-water binary mixture**

Note that, as the CMO is assumed, the flow rates of the *liquid* and the *vapour* phases for each section (either *rectifying* or *stripping*) are *constant*

The values of  $L$ ,  $V$ ,  $\bar{L}$ , and  $\bar{V}$  can be obtained by performing the material balances, as described and exemplified previously

From the above explanations and Figure 4.19, it illustrates that the *McCabe-Thiele technique* is a *powerful tool* to deal with the distillation problem, as it is *easy to use* and also *provides an insight* into the distillation column (on the *stage by stage* basis) *without* having to *measure* the values of the variables (*e.g.*, concentration, temperature, flow rate) *internally* (or *stage by stage*)

## 4.5 Open Steam Heating

The *open steam* heating is another technique used to heat up and vaporise the stream coming out of the bottom part of the distillation column; the *open steam heating* is used *in lieu* (ทดแทน) of the *re-boiler*

Let's consider the following Example, which illustrates the calculations of the distillation column using the *open steam heating* in replace of the *re-boiler*

**Example** The feed is 60 mol% MeOH and 40% water and is input as a *2-phase* mixture that flashes at 1 atm such that  $\frac{V_F}{F} = 0.3$

The feed flow rate is 350 kmol/h. The column is *well insulated* and has a *total condenser*

The reflux is returned to the column as a *saturated liquid*, and the external reflux ratio  $\left(\frac{L}{D}\right)$  is 3.0

The desired distillate and bottom concentrations are 95 mol% and 8 mol% MeOH

Instead of using a re-boiler, saturated steam at 1 atm is sparged ( $\Psi\Psi$ ) into the bottom of the column to provide the boil-up

Calculate the *total number of stages* and the *optimal feed location*

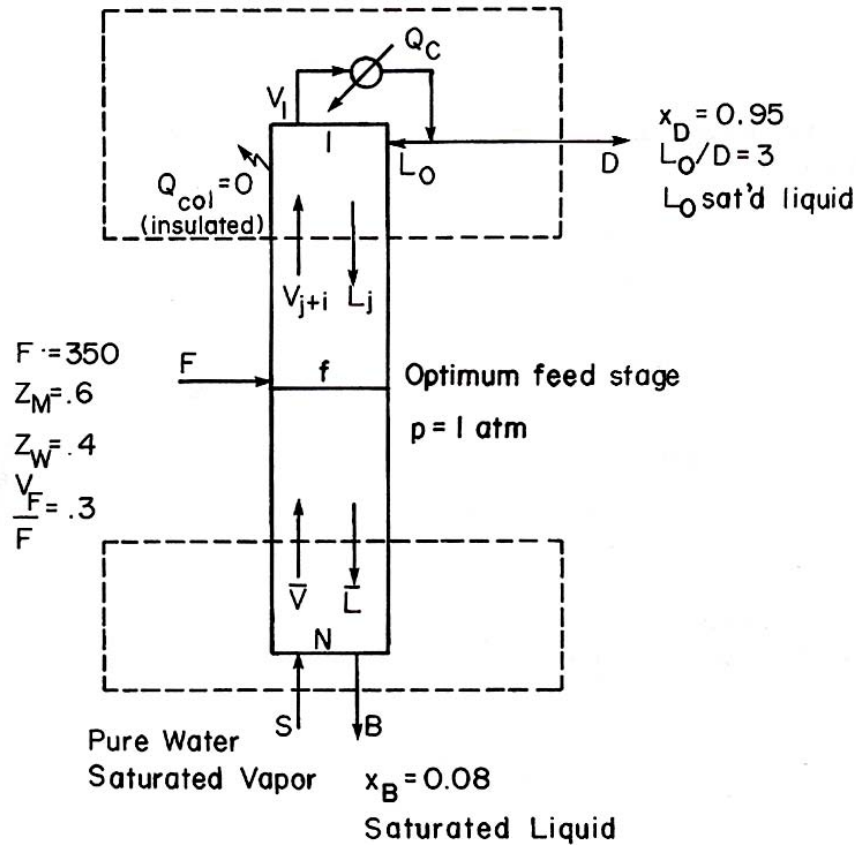
To check if the CMO is valid for this Example, we consider the latent heat of vaporisation of both components, which are as follows

$$\lambda_{\text{MeOH}} = 8.4 \text{ kcal/mol}$$

$$\lambda_{\text{water}} = 9.7 \text{ kcal/mol}$$

The **difference** is quite large (15-20%); thus the **CMO** should be used *with cautions* (ด้วยความระมัดระวัง)

The flow chart of this Example is shown in Figure 4.20



**Figure 4.20: A distillation column with the use of the open steam heating *in lieu* of the re-boiler** (from “Separation Process Engineering” by Wankat, 2007)

From the given reflux ratio  $\left(\frac{L}{D}\right)$ , the value of  $\frac{L}{V}$  can be calculated, using Eq. 4.45 as follows



$$\frac{L}{V} = \frac{\frac{L}{D}}{\frac{L}{D} + 1} = \frac{3.0}{3.0 + 1} = 0.75$$

Additionally, the *Y-intercept* of the *top* operating line can be computed as follows

$$\begin{aligned} \text{Y-intercept} &= \left(1 - \frac{L}{V}\right) x_D \\ &= (1 - 0.75)(0.90) \end{aligned}$$

$$\text{Y-intercept} = 0.24$$

It is given, in the problem statement, that

$$\frac{V_F}{F} = f = 0.3; \text{ thus}$$

$$\frac{L_F}{F} = q = 1 - f = 0.7$$

Hence, the *slope* of the *feed* line is

$$\text{Slope} = \frac{q}{q-1} = \frac{0.7}{0.7-1} = -2.33$$

In addition, the *feed* line must *intersect* with the  $y = x$  line at the point where  $x = y = z_F = 0.60$

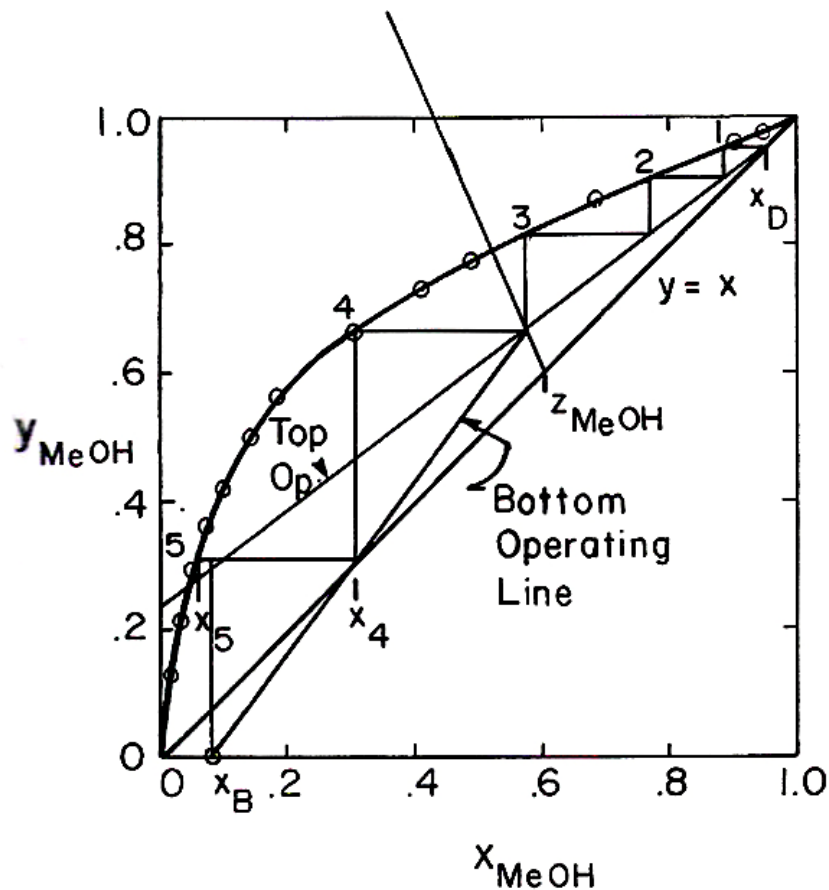
With the slope ( $= -2.33$ ) and the point of  $x = y = z_F = 0.60$ , the *feed* line can be *drawn*

Then, the *bottom* operating line can be drawn from the following *two* points:

- the *intersection* of the *top* operating line and the *feed* line
- the point on the  $y = x$  line at  $x_B = 0.08$

Eventually, the McCabe-Thiele diagram can be drawn as shown in Figure 4.21

The total *number of stages* = **5**, and the *optimal feed stage* is the stage **3**



**Figure 4.21: The McCabe-Thiele diagram for the open steam heating problem**

(from “Separation Process Engineering” by Wankat, 2007)

It is evident that the calculations for the open steam heating is similar to that of the distillation column using a re-boiler

Note that the total *number of stages* from the *computer simulation* is found to be **6** (the *difference* is due mainly to the *CMO assumption*)

## 4.6 General McCabe-Thiele Analysis Procedure

From the previous Examples, it is evident that, for any kinds/types of distillation problems, the column is normally divided into **2 sections**:

- the *enriching/rectifying* section or the section *above the feed*
- the *stripping* section or the section *below the feed*

and each section can be treated *separately* or is *independent* of another section

In fact, it is not necessary that the column has to be divided into 2 sections *only*; *in practical*, it is common that the *distillation column* is *divided* into *several* (more than 2) sections

This means that the column can have *more than single (one) feed* points

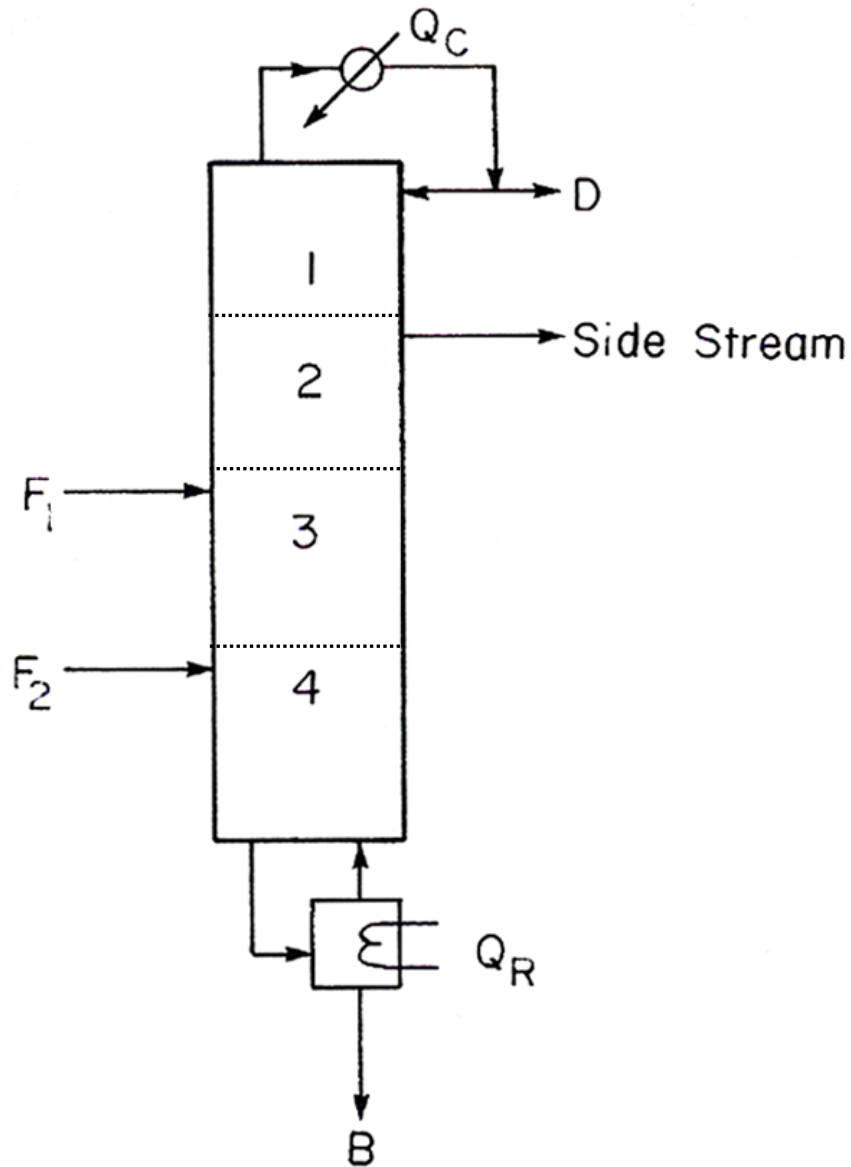
Additionally, this also means that the column can have *more output (exit) streams* than the *distillate (D)* and the *bottom (B)*

Consider Figure 4.22, which illustrates the distillation column with 2 feed streams ( $F_1$  and  $F_2$ ) and 3 output streams [distillate ( $D$ ), bottom ( $B$ ), and side stream ( $S$ )]

Note that the distillation column in Figure 4.22 is divided into 4 sections:

As mentioned earlier, *each* of these 4 sections can be treated *separately*

A standard procedure (or algorithm) for solving any distillation problems is as follows



**Figure 4.22: The distillation column with 2 feed streams and an additional side-stream exit stream**  
 (from “Separation Process Engineering” by Wankat, 2007)

- 1) *Draw a flow chart*, in which all known variables are labelled
- 2) Establish the equilibrium data (in the form of either data points or equation) and *draw/plot the equilibrium x-y diagram (curve)*
- 3) Check if the *CMO* is *valid* (by checking the latent heat of vaporisation of each component)
- 4) For *each section* (either *enriching* or *stripping* section),
  - a. locate a material balance envelope – keep in mind that the *fewer* the *streams/unknowns* involved, the *easier* the material balances would be



- b. establish the *overall* and *species balance equations*
  - c. write the *operating equations* by calculating all known slopes, intercepts, and intersections (mainly the intersections with the  $y = x$  line), and then *draw* the *operating line* for each section
- 5) Develop the *feed equations*: calculate  $q$  values, slopes, and  $y = x$  intersections, and then *draw* the *feed line* for each feed stream
- 6) For the operating and feed lines,
- a. try to plot as many operating and feed lines as possible

- b. if necessary, perform external material and energy balances (to obtain, *e.g.*, the values of  $D$  and/or  $B$ )
- 7) When all (or almost all) lines are in place (*i.e.* already drawn), *step off the stages* (ลากขั้นบันได), and determine the total *number of stages* and the *optimal feed location*

The following Example illustrates the calculations of the distillation problem using the standard procedure

**Example** Ethanol (EtOH) and water are to be separated in a distillation column, operating 1 atm, with a total condenser and partial re-boiler

There are 2 feed streams:

- Feed 1 is a *saturated vapour* and of the flow rate of 200 kmol/h, with 30 mol% EtOH
- Feed 2 is a *sub-cooled liquid*, in which 1 mole of vapour must be condensed inside the column in order to heat up 4 moles of the feed to its boiling point; the flow rate of Feed 2 is 300 kmol/h, with 40 mol% EtOH

The desired products are the *distillate* and the *bottom* with 72 and 2 mol% EtOH, respectively

The external reflux ratio is 1.0

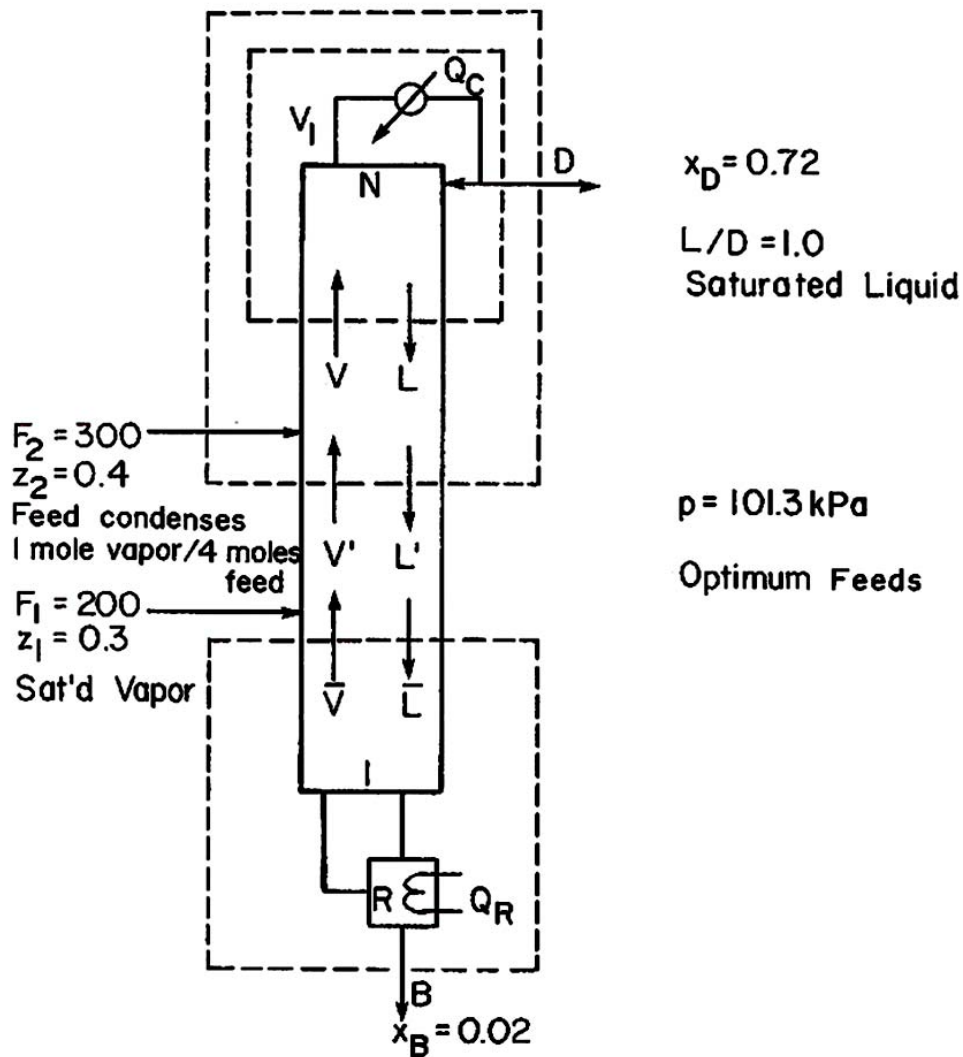
If all the feeds are to be input at their optimal locations, find the total number of stages and the optimal feed stages

The flow chart with all of known variables is depicted as Figure 4.23

Note that, in Figure 4.23, since the *concentration* of EtOH of *Feed 2* is *higher than* that of *Feed 1*, we place *Feed 2* higher than *Feed 1* (do you know why?)

The *top* operating line or the operating line for the top section – the *section* between the *top* of the column and *Feed 2* – is as follows

$$y = \frac{L}{V}x + \left(1 - \frac{L}{V}\right)x_D \quad (4.77)$$



**Figure 4.23: The distillation column with two feed streams**

(from “Separation Process Engineering” by Wankat, 2007)

The value of  $\frac{L}{V}$  can be computed from the given value of the reflux ratio  $\left(\frac{L}{D}\right)$  as follows

$$\frac{L}{V} = \frac{\frac{L}{D}}{\frac{L}{D} + 1} = \frac{1.0}{1.0 + 1} = 0.5$$

The *Y-intercept* of the *top* operating line (Eq. 4.77) can then be calculated as follows

$$\left(1 - \frac{L}{V}\right)x_D = (1 - 0.5)(0.72) = 0.36$$

With the *slope* of  $\frac{L}{V} = \mathbf{0.5}$  and *Y-intercept* of **0.36**, the *top operating line* can be drawn

Alternatively, this *top* operating line can be drawn when we have *2 points*: the *Y-intercept* and the *other* point

Normally, the **other point** is the *intersection* of the *operating* line and the  $y = x$  line

We have learned that the *top* operating line crosses the  $y = x$  line at the point where  $y = x = x_D$

Thus, in this Example, the *other point* is the point on the  $y = x$  line where  $y = x = x_D = 0.72$

To draw the *operating line* for the *middle section* (or the *middle operating line*) – note that the *middle section* is the section *between* the *two feed streams*, we perform the material balances from the *top* of the column to the *boarder*

between the *top* and the *middle* sections (the balance envelope can be seen as the *larger envelope* of the top section in Figure 4.23) as follows

Overall balance

$$F_2 + V' = L' + D \quad (4.78)$$

EtOH balance

$$z_{F_2} F_2 + y V' = x L' + x_D D \quad (4.79)$$

Note that  $L'$  and  $V'$  are the flow rates of the liquid and vapour streams of the *middle* section, respectively



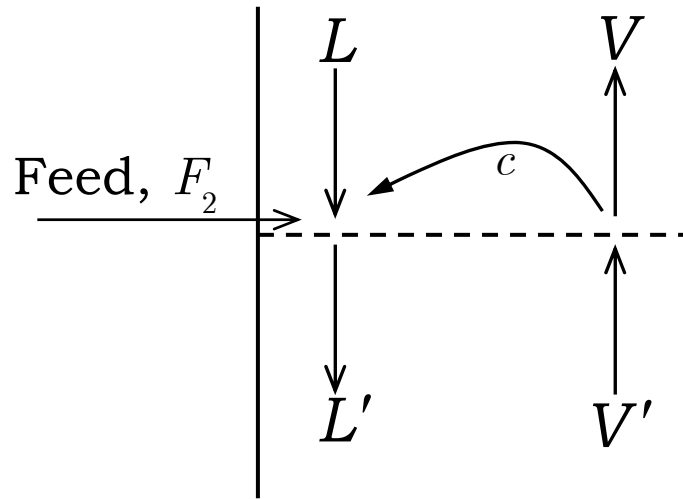
Re-arranging Eq. 4.79 gives

$$y = \frac{L'}{V'}x + \frac{(x_D D - z_{F_2} F_2)}{V'} \quad (4.80)$$

**Eq. 4.80** is the *operating line* for the *middle section* (for this Example)

Feed 2 (see the feed location of Feed 2 in Figure 4.23 on Page 109) is a *sub-cooled liquid*, in which 1 mol of vapour is to be condensed to heat up 4 mol of the feed to its boiling point

Let the amount of vapour condensed be denoted as  $c$



**Figure 4.24:** The condensation of vapour flowing from the middle section to the top section due to the fact that the Feed 2 is a sub-cooled liquid

In Figure 4.24, we obtain the following equations:

$$V' - c = V \quad (4.81)$$

$$L + F_2 + c = L' \quad (4.82)$$

Then, the value of  $q_{F_2}$  (the  $q$  value for Feed 2) can be obtained by adapting Eq. 4.36b to suit this situation as follows

$$q_{F_2} = \frac{L' - L}{F_2} \quad (4.83)$$

It is given that 1 mol of vapour is to be condensed to heat up 4 mol of the feed to its boiling point, which can be written in an equation form as follows

$$c = \frac{F_2}{4}$$

Thus, Eq. 4.82 becomes,

$$L + F_2 + \frac{F_2}{4} = L'$$

which can be re-arranged to

$$L' - L = F_2 + \frac{F_2}{4} = \frac{5}{4} F_2$$

Substituting  $L' - L = \frac{5}{4} F_2$  into Eq. 4.83 yields

$$q_{F_2} = \frac{5}{4} F_2 = 1.25$$

Accordingly, the *slope* of the *feed line* for **Feed 2** can be calculated as follows

$$\text{Slope}_{\text{Feed 2}} = \frac{q}{q-1} = \frac{1.25}{1.25-1} = 5.0$$

Additionally, the feed line of Feed 2 crosses the  $y = x$  line at the point where  $y = x = z_{F_2} = 0.4$

The value of  $L'$  can be computed using Eq. 4.83:

$$q_{F_2} = \frac{L' - L}{F_2} \quad (4.83)$$

as follows

$$L' = L + q_{F_2} F_2 \quad (4.84)$$

The liquid flow rate of the top section,  $L$ , can be calculated when the reflux ratio  $\left(\frac{L}{D}\right)$  and the distillate flow rate ( $D$ ) are known as follows

$$L = \left(\frac{L}{D}\right) D \quad (4.85)$$

To obtain the value of  $D$ , we must perform an *external* material balance, which results in

$$\begin{aligned} F_1 + F_2 &= D + B \\ (200) + (300) &= 500 = D + B \end{aligned} \quad (4.86)$$

$$z_{F_1}F_1 + z_{F_2}F_2 = x_D D + x_B B$$

$$\left[ (0.30)(200) \right] + \left[ (0.40)(300) \right] = 180 = (0.72)D + (0.08)B \quad (4.87)$$

Solving Eq. 4.86 & 4.87 simultaneously gives

$$D = 242.9 \text{ kmol/h}$$

Thus, the value of  $L$  can be computed using Eq. 4.85 as follows

$$\begin{aligned} L &= \left( \frac{L}{D} \right) D \\ &= (1.0)(242.9) \end{aligned}$$

$$L = 242.9 \text{ kmol/h}$$

Eventually, the value of  $L'$  can be computed using Eq. 4.84:

$$L' = L + q_{F_2}F_2 \quad (4.84)$$

as follows

$$L' = (242.9) + [(1.25)(300)] = 617.9 \text{ kmol/h}$$

Alternatively,  $L'$  can be calculated using Eq. 4.82:

$$L + F_2 + c = L' \quad (4.82)$$

as follows

$$L' = L + F_2 + c$$

$$L + F_2 + \frac{F_2}{4}$$

$$L' = 242.9 + 300 + \frac{300}{4}$$

$$L' = 617.9 \text{ kmol/h}$$

The value of  $V'$  can then be calculated from Eq. 4.78:

$$F_2 + V' = L' + D \quad (4.78)$$

as follows

$$\begin{aligned}V' &= L' + D - F_2 \\ &= 617.9 + 242.9 - 300 \\ V' &= 560.8 \text{ kmol/h}\end{aligned}$$

Accordingly, the value of  $\frac{L'}{V'}$  or the *slope* of the *middle* operating line (Eq. 4.80) is

$$\frac{L'}{V'} = \frac{617.9}{560.8} = 1.10$$

Additionally, the *Y-intercept*  $\left[ \frac{(x_D D - z_{F_2} F_2)}{V'} \right]$

of Eq. 4.80 can be computed as follows

$$\frac{(x_D D - z_{F_2} F_2)}{V'} = \frac{\{[(0.72)(242.9)] - [(0.4)(300)]\}}{(560.8)}$$

$$Y\text{-intercept (middle op. line)} = 0.098$$



With the *slope* and *Y-intercept*, the *middle* operating line can be drawn

Alternatively, since the ***middle* operating line** must **pass through** the **intersection** of the **top operating line** and the **feed line** of Feed 2 (in order for Feed 2 to be fed at the *optimal* location)

Thus, from the following two points:

- the *Y-intercept* of the *middle* operating line
- the *intersection* of the *top* operating line and the *feed line* of *Feed 2*

the *middle* operating line can be drawn

To draw the *feed line* for **Feed 1** is rather *straightforward*

Since **Feed 1** is a **saturated vapour**, its **feed line** is a *horizontal line*

Additionally, the feed line of Feed 1 must cross the  $y = x$  line at the point where  $y = x = z_{F_1} = 0.30$

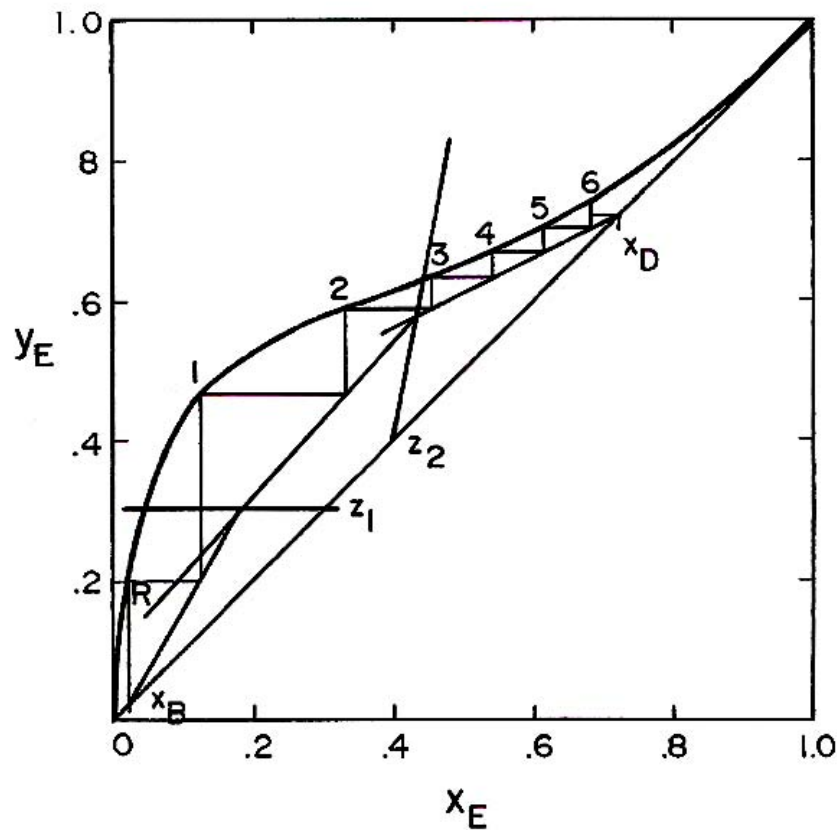
The *bottom* operating line, which can be written in an equation form as follows:

$$y = \frac{\bar{L}}{\bar{V}} x - \left( \frac{\bar{L}}{\bar{V}} - 1 \right) x_B \quad (4.88)$$

must pass through the *intersection* of the *middle* operating line and the *feed line* for *Feed 1*

Additionally, it crosses the  $y = x$  line at the point where  $y = x = x_B = 0.02$

Eventually, all operating and feed lines can be established, as shown in Figure 4.25



**Figure 4.25: The McCabe-Thiele diagram for the distillation problem with 2 feed streams**

(from “Separation Process Engineering” by Wankat, 2007)

From Figure 4.25, the number of stages is found to be 6.5 stages or 5.5 stages + 1 re-boiler, and the optimal locations are

- stage 1 (above the re-boiler) for Feed 1
- stage 2 (above the re-boiler) for Feed 2

## 4.7 Other Distillation Column Situations

Some *special* cases or situations of the operation of the distillation column are summarised as follows

### 4.7.1 Partial condenser

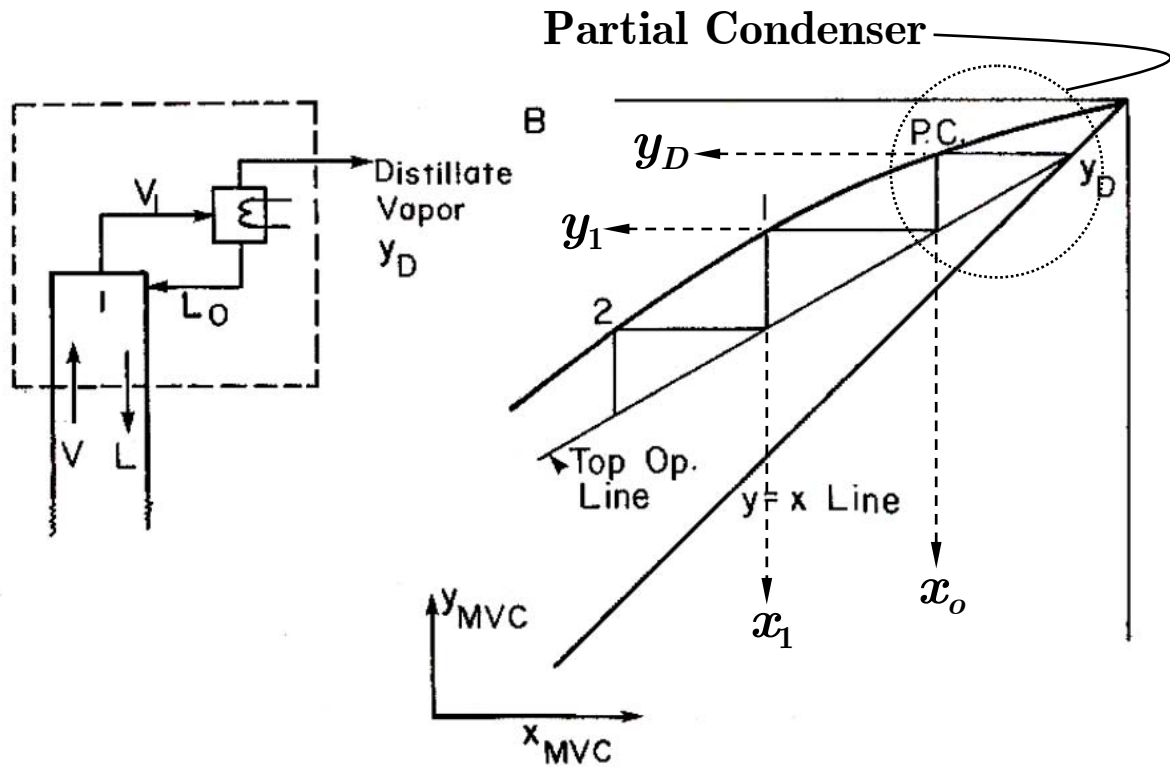
In the *normal* distillation, the condenser used in the operation is a *total* condenser, at which *all vapour* from the top of the column is *condensed* to *liquid*

In the *total* condenser, both *distillate* ( $D$ ) and *reflux* ( $L_o$ ) are *liquid*

On the contrary, a *partial* condenser ***condenses*** only a ***fraction of vapour*** from the top of the column

In the *partial* condenser, the *distillate* is *vapour*, while the *reflux* is *liquid*, as illustrated in Figure 4.26

In Figure 4.26, it is evident that there is an *equilibrium* between *vapour* (*distillate*:  $D$ , with the concentration of  $y_D$ ) and *liquid* (*reflux*:  $L_o$ , with the concentration of  $x_o$ ) at the condenser



**Figure 4.26: A partial condenser**

(from “Separation Process Engineering” by Wankat, 2007)

Thus,

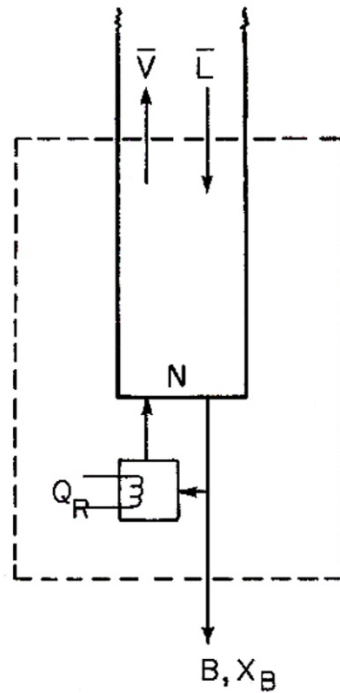
- the concentration of the distillate ( $y_D$ ) is no longer equal to the concentration of the reflux ( $x_o$ ), but  $x_o$  can be read from the equilibrium curve when  $y_D$  is known
- the *partial condenser* is considered an *additional equilibrium stage*

## 4.7.2 Total Re-boiler

The re-boiler commonly used in the distillation operation is a *partial* re-boiler, in which only a *fraction* of *liquid* flowing out of the last stage of the distillation column is vaporised (see Figure 4.4 on Page 9)

Hence, there is an *equilibrium* between the *liquid* (that flowing out of the last stage) and *vapour* (that is vaporised in the re-boiler and thus returned back to the column); this is why a *re-boiler* is considered an *additional stage*

In the ***total*** re-boiler, ***all liquid*** flowing into the re-boiler is completely ***vaporised***, as depicted in Figure 4.27



**Figure 4.27: A total re-boiler**

(from “Separation Process Engineering” by Wankat, 2007)

Note that the bottom ( $B$ ) with the concentration of  $x_B$  is separated from the liquid stream from the last stage of the column before entering the re-boiler; it is not separated at the re-boiler as *per* the partial re-boiler



Since *all* the *liquid* flowing into the re-boiler is *vaporised* and there is *no reaction* at the re-boiler, the *concentration* of the *vapour* coming out of the re-boiler is equal to that of the liquid flowing in

This means that there is *no equilibrium* between the *vapour* and the *liquid* phases at the *re-boiler*

Accordingly, the ***total*** re-boiler **cannot** be **considered** as an ***additional stage*** as *per* the case of *partial* re-boiler

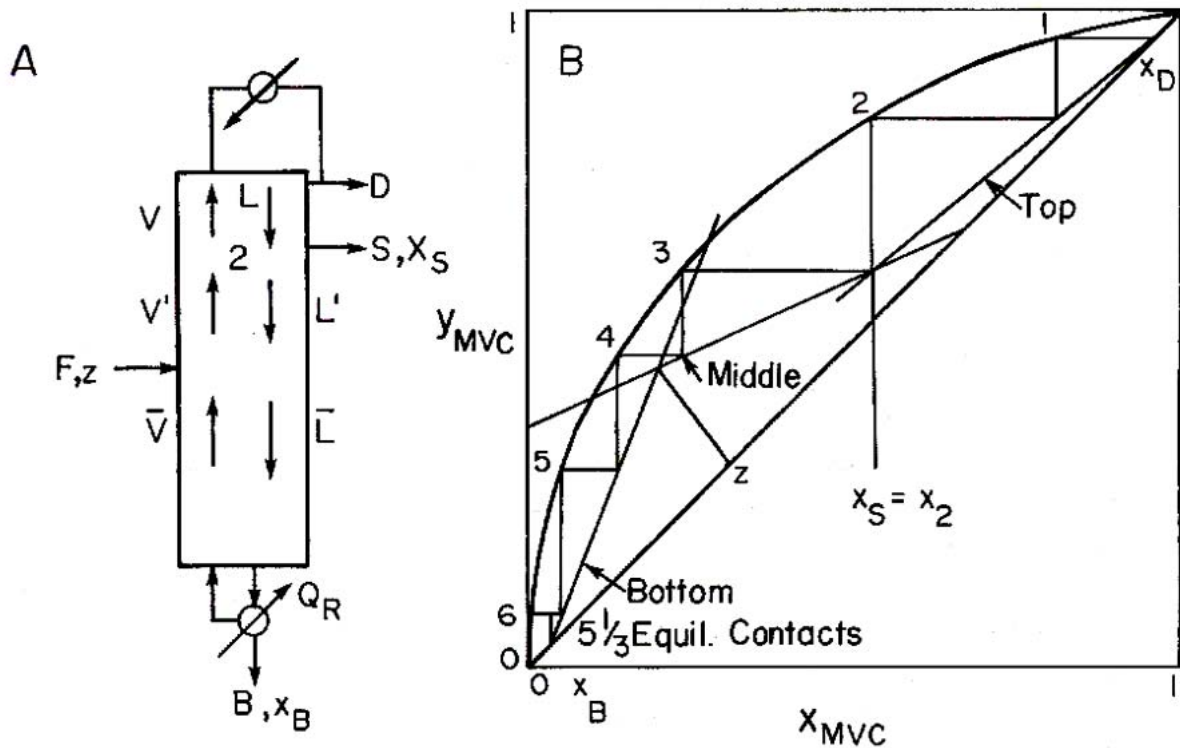
### 4.7.3 Side streams or withdrawal lines

As mentioned earlier, in the real distillation operation, there are more than the distillate ( $D$ ) and the bottom ( $B$ ); there are also additional output streams, which are commonly called “**side streams**” and “**withdrawal lines**”

When there are *side streams* or *withdrawal streams*, for each stream, there are 3 *additional variables* associated with that stream, comprising

- its flow rate,  $S$
- its composition (concentration,  $x_s$ )
- its withdrawal stage

Consider the distillation column with a side stream and the corresponding McCabe-Thiele diagram in Figure 4.28



**Figure 4.28: The distillation column with a side stream and the corresponding McCabe-Thiele diagram**

(from “Separation Process Engineering” by Wankat, 2007)

In Figure 4.28, the column comprises 3 sections:

- the *top* section: the section above the side stream (including the condenser)
- the *middle* section: the section *between* the *side stream* and the *feed*

- the *bottom* section: the section *below* the *feed* stage

To obtain the *operating* equation (line) for each section, we have to perform material balances with the appropriate balance envelope

Note that the *side stream* can be treated as a *feed*, but in the *opposite direction*

The *middle* operating line equation of Figure 4.28, which involves the side stream, can be written as follows (try to derive it yourself)

$$y = \frac{L'}{V'} x + \frac{x_D D + x_S S}{V'} \quad (4.89)$$

Note that the *middle* operating line crosses the  $y = x$  line at the point where

$$y = x = \frac{x_D D + x_S S}{D + S} \quad (4.90)$$

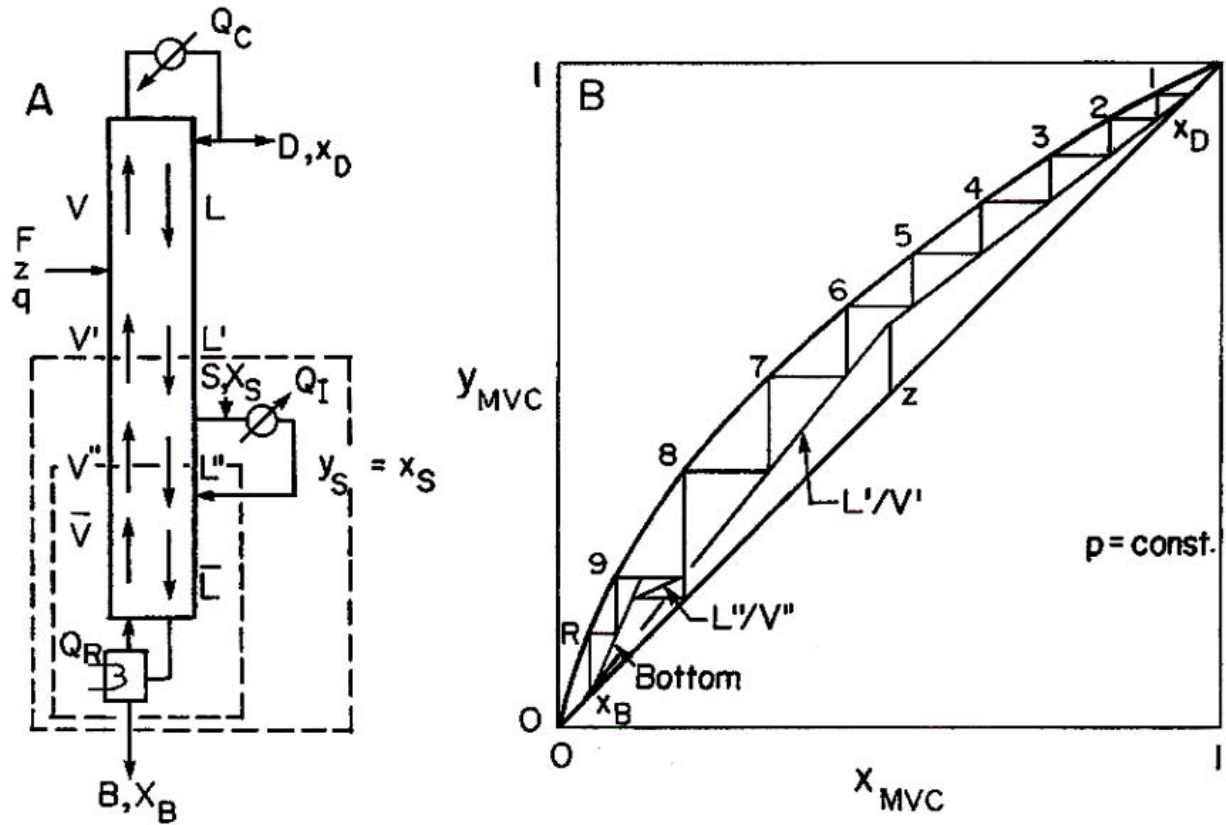
As mentioned recently, the *side stream* can be treated as a *feed*; hence, if

- the side stream is a *saturated liquid*, the operating line of the side stream is a *vertical* line with concentration of  $x_S$
- the side stream is a *saturated vapour*, the operating line of the side stream is a *horizontal* line with the concentration of  $y_S$

#### 4.7.4 Intermediate re-boiler and intermediate condenser

As there are side streams, in addition to the condenser at the top and the re-boiler at the bottom of the column, there may be condensers and re-boilers associated with the *side streams*, which is generally called *intermediate condensers* and *intermediate re-boilers*, respectively

An *intermediate re-boiler* removes a *liquid* side stream from the column, boils (or vaporises), and re-injects it (as a *vapour*) back to the same column, as illustrated in Figure 4.29



**Figure 4.29: A distillation column with the intermediate re-boiler**

(from “Separation Process Engineering” by Wankat, 2007)

Since there is an *additional* re-boiler added into the column, an *additional* energy requirement (*i.e.* heating load) of  $Q_I$  is required

As  $x_S > x_B$  (do you know why?), it is easier for this liquid side stream to be vaporised; thus, the heat requirement (or the heating load) is lower than that of the *bottom* re-boiler

Additionally, by having an *intermediate* re-boiler, it *reduces* the *heating load* of the *bottom* re-boiler

An *intermediate condenser* removes a *vapour* side stream from the column, condenses it, and re-injects it (as a liquid) back to the same column

Since the *vapour* side stream contains more amounts of *less volatile* components than does the distillate



These *less volatile* components can be *condensed to liquid easier* than the *more volatile* components

Thus, for the *intermediate condenser*, the *coolant with a higher temperature than* that used at the *top condenser* can be used; in other words, by having the *intermediate condenser*, it *reduces* the *cooling* load at the *top condenser*

It is important to note that the optimal feed stage for the *re-injecting* stream (for either intermediate re-boiler or intermediate condenser) is the stage *adjacent* (ที่อ้อยติดกัน) to the *withdrawal* stage

### 4.7.5 Stripping and enriching columns

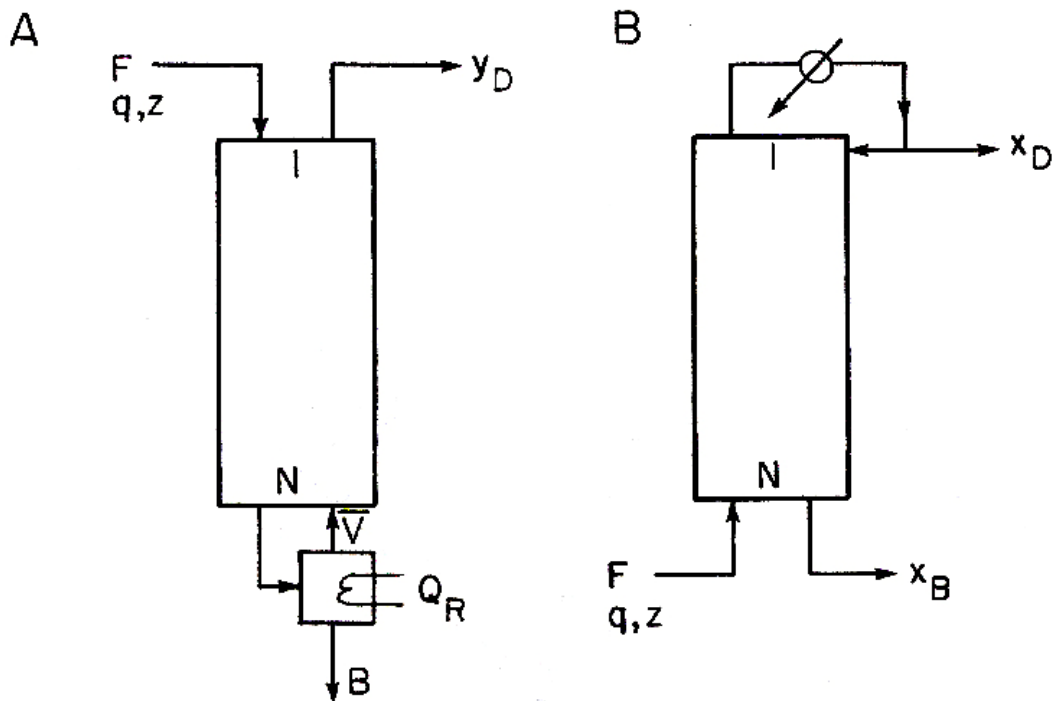
A *stripping* column is a distillation column that has *only* a *re-boiler* with the feed fed into the column from the top

For a *stripping* column, the *feed* must be *liquid* (can be either *saturated* or *sub-cooled*)

An *enriching* column is a distillation column that has *only* the *condenser* with the feed introduced to the column from the bottom

The *feed* of the *enriching* column has to be vapour, which can be either *saturated* or *super-heated* vapour

Figure 4.30 depicts the *stripping* (A) and *enriching* (B) columns



**Figure 4.30: A stripping column (A) and an enriching column (B)**

(from “Separation Process Engineering” by Wankat, 2007)

In a *stripping* column, the *bottom* can be *very pure*, but the vapour product is not

Hence, the *main purpose* of using the *stripping* column is to obtain the *bottom* with as *high concentration* of a *less volatile* component as possible

On the other hand, in the *enriching* column, the *distillate* is *very pure*, but the liquid product (the bottom) may not be so

Accordingly, the *enriching* column is used when we want to obtain the *distillate* with the *high concentration* of a *more volatile* component

The *operating line* equations of the *stripping* column are as follows:

- The *top operating line* equation:

$$y = y_D$$

- The *bottom operating line* equation:

$$y = \frac{\bar{L}}{\bar{V}} x - \left( \frac{\bar{L}}{\bar{V}} - 1 \right) x_B \quad (4.88)$$

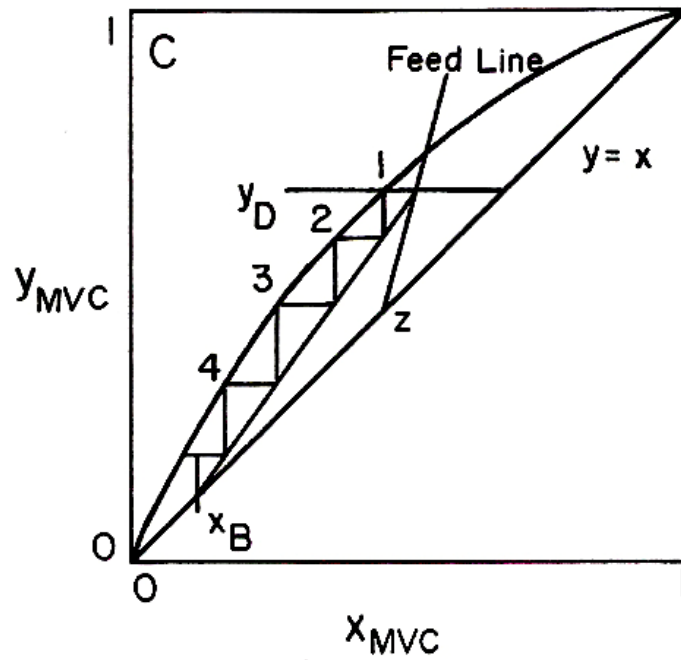
which is, in fact, the operating equation for the *stripping section* of the *full* distillation column

These operating lines are as illustrated in Figure 4.31

The *operating line* equation for the *enriching* column can be written as follows:

- The *top operating line*:

$$y = \frac{L}{V} x + \left( 1 - \frac{L}{V} \right) x_D \quad (4.77)$$



**Figure 4.31: The operating lines for a stripping column**

(from “Separation Process Engineering” by Wankat, 2007)

- The *bottom* operating line:

$$x = x_B$$

Note the *top operating line* equation of the enriching column is, in fact, the *operating line* equation of the *enriching section* of the *full* distillation column

## 4.8 Limiting Operating Conditions

There are 2 *limiting* conditions for distillation operations:

- *Minimum reflux*  $\left(\frac{L}{D}\right)_{\min}$ , which is the condition that the external **reflux ratio** is as **minimal** as possible; in this condition, we need an **infinite** ( $\infty$ ) number of stages to obtain a desired separation
- *Total reflux*, which is the condition that **all overhead vapour** is **condensed** and **re-turned** to the *column* as a **reflux** (*i.e.* no distillate), and **all bottom liquid** is **boiled** at the *re-boiler* and **re-turned** to the *column* (*i.e.* no bottom product)

For the case of **total reflux**, as illustrated in Figure 4.32, the steady-state **feed rate** must be *zero*, with  $\frac{L}{V} = \frac{\bar{L}}{\bar{V}} = 1.0$  (do you know why?)

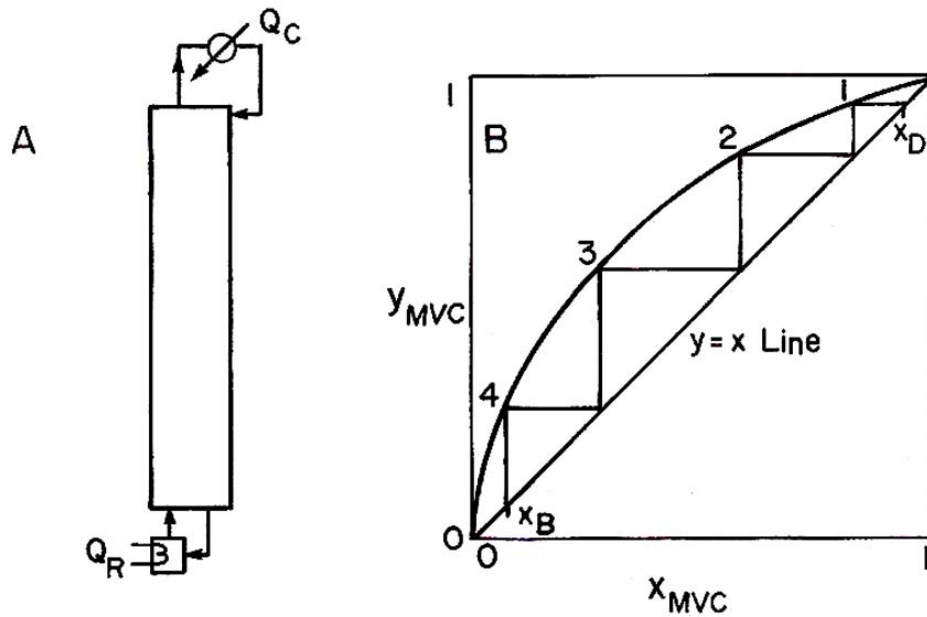
Also for this case, the **operating lines** of both sections (*enriching* and *stripping*) are

$$y = x$$

The **total reflux** condition is used for **starting up** the column or for **determining** the column **efficiency**

With this condition, it yields the *minimum* **number of stages**



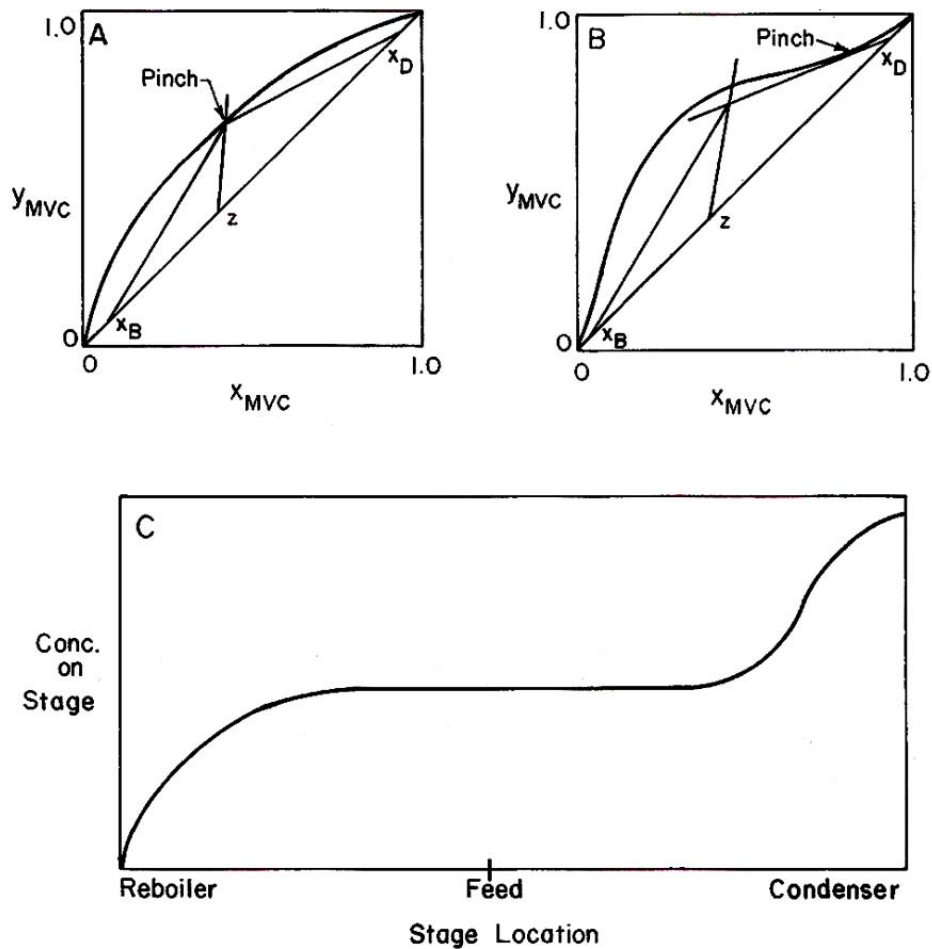


**Figure 4.32: The total reflux distillation condition**

(from “Separation Process Engineering” by Wankat, 2007)

For the case of **minimum reflux**, the **reflux** and the **reflux ratio** is **minimal**, and the **number of stage** required is **infinite** ( $\infty$ )

To obtain these, the **top operating line** must **touch** the *equilibrium line*, as illustrated in Figure 4.33



**Figure 4.33: The McCabe-Thiele diagram and the concentration profile of the minimum reflux ratio condition**

(from “Separation Process Engineering” by Wankat, 2007)

Normally, the *touching point* is the point where the **top operating line** *intersects* with the **feed line**, and that *intersection* has to be **on** the *equilibrium line*

However, it is *impossible* that the *operating line* (of the *more volatile* component) be *higher* than the *equilibrium line*; thus, in some cases (generally, the case that the mixture has an *azeotropic* point), the *touching point* is as illustrated in the *right Figure* in Figure 4.33

Note that, the *concentration profile* for the *minimum reflux* case is as shown in the *lower Figure* in Figure 4.33; the *concentration* around the *touching point* is found to be nearly *constant*

The point where the operating line(s) *touch* the equilibrium line or the touching point is commonly called the “**pinch**” point

The *slope*  $\left(\frac{L}{V}\right)$  of the *top operating line* in the case of *minimum reflux ratio* is **minimal** and denoted as  $\left(\frac{L}{V}\right)_{\min}$

Once  $\left(\frac{L}{V}\right)_{\min}$  is obtained, the value of  $\left(\frac{L}{D}\right)_{\min}$  or the ***minimum reflux ratio*** can then be calculated from the value of  $\left(\frac{L}{V}\right)_{\min}$  as follows

$$\left(\frac{L}{D}\right)_{\min} = \frac{\left(\frac{L}{V}\right)_{\min}}{1 - \left(\frac{L}{V}\right)_{\min}} \quad (4.91)$$

The minimum reflux ratio or  $\left(\frac{L}{D}\right)_{\min}$  is used to determine the *optimal* operating reflux ratio

Normally, the *optimal* operating reflux ratio is between 1.05-1.25 times  $\left(\frac{L}{D}\right)_{\min}$

## 4.9 Efficiencies

Commonly, there are **2 types** of **efficiencies**:

- *Overall* efficiency
- *Murphree* efficiency

The *overall efficiency*, denoted as  $E_o$ , is defined as the *ratio* of the number of *theoretical equilibrium* stages obtained from the calculations (using, *e.g.*, the McCabe-Thiele diagram) to the *actual* number of stages used in the real distillation operation, which can be written in an *equation* form as follows:

$$E_o = \frac{N_{\text{equilibrium}}}{N_{\text{actual}}} \quad (4.92)$$

This type of efficiency (*i.e.* overall efficiency) is based on the concept that, in fact, each stage has not reached its equilibrium yet, and more stages are required (to reach the equilibrium)

Hence, the meaning of  $N_{\text{actual}}$  is the *actual* number of stages needed to reach the *equilibrium* between the *liquid* and the *vapour* phases

For example, if the *overall* efficiency is **50%**, it means that we *need 2 actual* stages for *each theoretical* equilibrium stage to *allow* the *liquid* phase and the *vapour* phase to reach the *equilibrium*

For the *Murphree* efficiency, it can be divided into 2 types:

- Murphree *vapour* efficiency ( $E_{MV}$ ):

$$E_{MV} = \frac{y_i - y_{i+1}}{y_i^* - y_{i+1}} \quad (4.93)$$

- Murphree *liquid* efficiency ( $E_{ML}$ ):

$$E_{ML} = \frac{x_i - x_{i-1}}{x_i^* - x_{i-1}} \quad (4.94)$$

where  $x_i^*$  and  $y_i^*$  are the concentration of species  $i$  (normally the *more volatile* components) at *equilibrium* of the liquid and vapour phases, respectively

By considering both Eqs. 4.93 and 4.94, the *concept* of the *Murphree* efficiency is similar to that of the *overall* efficiency, but it considers the *efficiency in each phase (i.e. either liquid or vapour phase) separately*

The *similarity* of both efficiencies is that it is based on the concept that, in the real operation, each stage does not reach the equilibrium of the liquid and the vapour phases (which results in the fact that  $x_i < x_i^*$  and  $y_i < y_i^*$ ); thus, additional stages are needed