Chapter 8: Absorption and Stripping

In addition to the distillation, there are other unit operations used for separating substances.

Absorption is the unit operation in which one or more components of a gas stream are removed from the gas mixture by being absorbed onto a non-volatile liquid (called a “solvent”)

In this case, the solvent is the separating agent.
**Stripping** is the operation that is in the opposite direction to the **absorption**, in which one or more **gaseous** components in a **liquid** stream is **removed** from the **gas-liquid** solution by being **vaporised** into an **insoluble gas** stream.

In the **stripping** operation, the **insoluble gas stream** is the **separating agent**.

**What is the separating agent in the distillation?**
Absorption can be

- **physical**, when the *solute* is *dissolved* into the *solvent* because it has *higher solubility* in the solvent than other gases

- **chemical**, when the *solute* *reacts with the solvent*, and the resulting product still remains in the solvent

Normally, a *reversible* reaction between the solute and the solvent is preferred, in order for the solvent to be regenerated

(นำกลับมาใช้ใหม่)

Similar to the *distillation*, both *absorption* and *stripping* are operated as *equilibrium* stage operations, in which *liquid* and *vapour* are *in contact* with each other
However, in both absorption and stripping operations, the columns are simpler than the distillation column; normally, they do **not need condensers and re-boilers** as per the distillation.

An example of the uses of absorption and **stripping** is a **gas treatment plant**, shown in Figure 8.1.

The **mixture** of the gases (i.e. the gas to be treated in Figure 8.1), from which one component (e.g., CO₂ or H₂S) in the gas mixture needs to be removed, is passed through an **absorber**, in which a **liquid-phase** solvent (e.g., MEA or amine solvent in water) is passed though, to absorb that gaseous component.
The removal component is mixed (physically or chemically) with the solvent

The resulting solvent is heated by a heater and becomes the saturated solvent (so that the resulting solvent can be stripped)
The resulting \textit{saturated} solvent is, subsequently, passed through a \textit{stripper} (or a \textit{stripping column}), in which the \textit{gaseous} component in the \textit{saturated} solvent is \textit{stripped off} by a \textit{stripping gas} (\textit{e.g.}, steam)

In this Example (Figure 8.1), the \textit{desired product}

- for an absorber is a \textit{treated gas} stream
- for a stripper is a \textit{recycle solvent} stream
8.1 Absorption and Stripping Equilibria

Absorption and stripping involves, at least, 3 components (as illustrated recently) and 2 phases

Generally, for simplicity, we often assume the following:

- **Carrier gas** is insoluble (into a solvent)
- **Solvent** is non-volatile (thus, the loss of solvent due to vaporisation is negligible)
- The system is isothermal (constant temperature) and isobaric (constant pressure)

Since the absorption or stripping system comprises 3 species and 2 phases, the degree of freedom ($F$), calculated using the Gibbs phase rule, is
\[ F = C - P + 2 = 3 - 2 + 2 = 3 \]

As we assume that the system is

- *isothermal*, which results in the fact that the system’s \textit{temperature} is \textit{constant}
- *isobaric*, which results in the fact that the system’s \textit{pressure} is \textit{constant}

the degree of freedom \((F)\) is reduced to one \((1)\)

Normally, an \textit{equilibrium data} is used as the \textit{remaining} degree of freedom \((F)\)

In general, the \textit{amount} of \textit{solute} (\textit{i.e.} a gas to be removed from the gas mixture) is relatively \textit{low}
For a low concentration of a solute, a Henry’s law is employed to express the equilibrium between the concentration (e.g., mole fraction or percent) of the solute in the gas phase and that in the liquid phase as follows

\[ P_B = H_B x_B \]  

(8.1)

where

\[ P_B = \text{partial pressure of the solute B in the gas phase (or in the gas mixture)} \]

\[ x_B = \text{concentration (in mole fraction) of the solute B in the liquid phase} \]

\[ H_B = \text{Henry’s constant for the solute B} \]

A mole fraction of the solute B in the gas phase \( (y_B) \) can be described by the following equation:
\[ y_B = \frac{P_B}{P_{total}} \quad (8.2) \]

which can be re-arranged to

\[ P_B = y_B P_{total} \quad (8.3) \]

Combining Eq. 8.3 with Eq. 8.1 and re-arranging the resulting equation gives

\[ y_B = \frac{H_B}{P_{total}} x_B \quad (8.4) \]

As we have assumed previously that the system is isobaric [which results in the fact the system’s pressure (\( P_{total} \)) is constant], Eq. 8.4, which is an equilibrium equation for absorption and stripping operations, is a linear relationship with the slope of \( \frac{H_B}{P_{total}} \).
8.2 Operating Lines for Absorption

We have learned previously from the distillation operation that if the operating line is linear, it would be easy to use.

To have a linear or straight operating line, it is required that

- the energy balance is automatically satisfied
- both liquid and gas (or vapour) flow rates are constant

In order for the energy balance to be automatically satisfied, we have to assume that
• the heat of absorption is negligible
• the operation is isothermal
• a solvent is non-volatile
• a carrier gas is insoluble (into the solvent)

For any gas absorber, as illustrated in Figure 8.2, by employing the above assumptions, we obtain the facts that

\[ L_1 = L_2 = \ldots = L_N = L \]  \hspace{1cm} (8.5)
\[ G_N = G_{N-1} = \ldots = G_1 = G \]  \hspace{1cm} (8.6)

It is important to note that, to make the liquid and gas flow rates constant, especially for the concentrated mixture (of either liquid or gas phase), we cannot use the overall or total flow rates of gas and liquid.
In other words,

- the **liquid** flow rate \((L)\) must be the molar or mass flow rate of a **non-volatile solvent**
- the **gas** flow rate \((G)\) must be the molar or mass flow rate of an **insoluble carrier gas**
Accordingly, the *mole fractions* of solute B in the gas \( (y_B) \) and the liquid \( (x_B) \) phases, which are defined as

- \( y_B = \frac{\text{moles of solute B (in the gas phase)}}{\text{total moles of a gas mixture}} \)

or

\[
y_B = \frac{\text{moles of solute B (in the gas phase)}}{\text{moles of carrier gas A} + \text{moles of solute B}}
\]

and

- \( x_B = \frac{\text{moles of solute B (in the liquid phase)}}{\text{total moles of a solution}} \)

or

\[
x_B = \frac{\text{moles of solute B (in the liquid phase)}}{\text{moles of a solvent} + \text{moles of solute B}}
\]

have to be modified to
\[ Y_B = \frac{\text{moles of solute B}}{\text{moles of pure carrier gas A}} \]

\[ X_B = \frac{\text{moles of solute B}}{\text{moles of pure solvent}} \]

The relationships between \( Y_B \) and \( y_B \) and between \( X_B \) and \( x_B \) can be written as follows

\[ Y_B = \frac{y_B}{1 - y_B} \quad (8.7) \]

\[ X_B = \frac{x_B}{1 - x_B} \quad (8.8) \]

Thus, by performing material balances for the given envelope in Figure 8.2, we obtain the following equation:

\[ GY_{j+1} + LX_o = GY_1 + LX_j \quad (8.9) \]
Re-arranging Eq. 8.9 for $Y_{j+1}$ results in

$$ Y_{j+1} = \frac{L}{G} X_j + \left( Y_1 - \frac{L}{G} X_o \right) \quad (8.10) $$

Eq. 8.10 is an operating line for absorption, with

- the slope of $\frac{L}{G}$
- the Y-intercept of $Y_1 - \frac{L}{G} X_o$

In order to obtain the number of equilibrium stages required for the absorption operation from the initial concentration of $X_o$ to the final concentration of $X_N$, the following procedure, which is similar to that for the distillation operation, is employed
1) Draw an *equilibrium* line on the $X$-$Y$ coordinate.

2) Draw an *operating* line; the values of $X_o$, $Y_{N+1}$, $Y_1$, and $\frac{L}{G}$ are generally known – note that the point $\left(X_o, Y_1\right)$ must be on the operating line.

3) Locate the point $\left(X_o, Y_1\right)$ and step off stages between the *operating* line and the *equilibrium* line from $X = X_o$ until it reaches the final concentration of $X_N$.

This $X$-$Y$ diagram, which consists of the *operating* and *equilibrium* lines for the *absorption* operation, is still called the “McCabe-Thiele diagram”
An example of the *McCabe-Thiele* diagram for *absorption* is illustrated in Figure 8.3

![McCabe-Thiele diagram](image)

**Figure 8.3: A McCabe-Thiele diagram for the absorption operation**

(from “Separation Process Engineering” by Wankat, 2007)

Note that the *operating* line in the McCabe-Thiele diagram for *absorption* is *above* the *equilibrium* line
This is because the solute is transferred from the gas phase (i.e. the gas mixture) to the liquid phase (i.e. the solvent), which is in the opposite direction to the distillation operation, in which the material (or the more volatile component: MVC) is transferred from the liquid phase to the gas phase.

The dotted lines in Figure 8.3 is the minimum $\frac{L}{G}$ line or the $\left(\frac{L}{G}\right)_{\text{min}}$ line; it is the line drawing from the initial point of $(X_o, Y_1)$ until touching the equilibrium line at the point where $Y = Y_{N+1}$ (note that $Y_{N+1}$ is the concentration of the solute in the inlet gas stream)
It is important to note that, if the system is **NOT isothermal** (as in the case of the *distillation* operation), the *operating* line is still *linear*, but the *equilibrium* line is *no longer* linear.

The following Example illustrates how to calculate the required **number** of *equilibrium stages* for the *absorption* operation.
Example  A gas stream containing 90 mol% N$_2$ and 10% CO$_2$ is passed through an absorber, in which pure and cool water at 5 °C is used as a solvent.

The operation is assumed to be isothermal at 5 °C and isobaric at 10 atm.

The liquid flow rate is 1.5 times the minimum liquid flow rate \[
\frac{L}{G}_{\text{min}}
\]

Determine the number of equilibrium stages required to absorb 92 mol% of CO$_2$.

Given Henry’s constant of CO$_2$ in water at 5 °C of 876 atm/mole fraction.
Basis: 100 mol/h of the gas mixture

The schematic diagram for this absorption operation can be illustrated as follows

(from “Separation Process Engineering” by Wankat, 2007)

Substituting given numerical values into Eq. 8.4:

\[
y = \frac{H}{P_{\text{total}}} x
\]  

(8.4)
gives

\[ y = \frac{876}{10} x = 87.6x \]  

(8.11)

Eq. 8.11 is the equilibrium equation; thus, the equilibrium or the \( y \)-\( x \) data can be computed and summarised in the following Table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.00876</td>
</tr>
<tr>
<td>0.0004</td>
<td>0.0350</td>
</tr>
<tr>
<td>0.0006</td>
<td>0.0526</td>
</tr>
<tr>
<td>0.0008</td>
<td>0.0701</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.0876</td>
</tr>
<tr>
<td>0.0012</td>
<td>0.1051</td>
</tr>
</tbody>
</table>

However, both \( x \) and \( y \) values have to be converted to \( X \) and \( Y \) as exemplified on the next Page
• For $x = 0.0001$:

$$X = \frac{x}{1 - x} = \frac{0.0001}{1 - 0.0001} \approx 0.0001$$

• For $y = 0.00876$:

$$Y = \frac{y}{1 - y} = \frac{0.00876}{1 - 0.00876} \approx 0.00884$$

Thus, the equilibrium data on $Y$-$X$ co-ordinate are as follows

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.00884</td>
</tr>
<tr>
<td>0.0004</td>
<td>0.0363</td>
</tr>
<tr>
<td>0.0006</td>
<td>0.0555</td>
</tr>
<tr>
<td>0.0008</td>
<td>0.0754</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.0960</td>
</tr>
<tr>
<td>0.0012</td>
<td>0.1175</td>
</tr>
</tbody>
</table>
Since the basis of calculation is 100 mol/h of the gas mixture, the amount in “mole” of CO₂ in the *input* stream is

\[
\left( n_{\text{CO}_2}\right)_{\text{inlet}} = \left( \frac{10}{100} \right) \times 100 \text{ mol/h} = 10 \text{ mol/h}
\]

The amount of carrier gas (*i.e.* N₂ in this Example) is given as 90%, meaning that the **flow rate** of the *carrier gas (only)* is

\[
n_{N_2} = \left( \frac{90}{100} \right) \times 100 \text{ mol/h} = 90 \text{ mol/h}
\]

From the problem statement, it is required that 92 mol% of CO₂ is absorbed by the solvent (*i.e.* cool water); this means that only 8% of CO₂ flowing into the absorber remains in the gas mixture.
Accordingly, the amount in “mole” of CO$_2$ in the outlet gas stream can be calculated as follows

\[
\left( n_{CO_2} \right)_{\text{outlet}} = \left( \frac{8}{100} \right) \times \left( n_{CO_2} \right)_{\text{inlet}}
\]

\[
\left( n_{CO_2} \right)_{\text{outlet}} = \left( \frac{8}{100} \right) \times 10 \text{ mol/h} = 0.8 \text{ mol/h}
\]

Hence, the inlet concentration (in “mole ratio”) of CO$_2$ for the gas phase $\left( Y_{N+1} \right)$ is

\[
Y_{N+1} = \left( \frac{n_{CO_2}}{n_{N_2}} \right)_{\text{inlet}} = \frac{10}{90} = 0.11
\]

and the outlet concentration (in “mole ratio”) of the CO$_2$ in the gas phase $\left( Y_1 \right)$ is

\[
Y_1 = \left( \frac{n_{CO_2}}{n_{N_2}} \right)_{\text{outlet}} = \frac{0.8}{90} = 0.0089
\]
Since pure water is used as the solvent, it results in the fact that \( x_o = 0 \), which means that
\[
X_o = 0
\]

Accordingly, the origin of the operating line is the point \((X_o, Y_1)\) of \((0, 0.0089)\).

The equilibrium line (from the equilibrium data on Page 24) can be plotted on the \( Y-X \) coordinate as shown on Page 28 (as a solid line).

The \( \left( \frac{L}{G} \right)_{\text{min}} \) line is the line originates from the point of \((0, 0.0089)\) and touches the equilibrium line at \( Y = Y_{N+1} = 0.11 \); note that the \( X \) value at the touching point can be read as 0.00105.
The slope of the \(\frac{L}{G}\) subscripts \(\text{min}\) line (the dotted lines) is found to be 96.3

\[
\frac{\Delta Y}{\Delta X} = \frac{(0.11 - 0.0089)}{(0.00105 - 0)}
\]

Thus, the actual slope of the operating line is

\[1.5 \times \left(\frac{L}{G}\right)_{\text{min}} = 1.5 \times 96.3 = 144.5\]
The origin of the *actual* operating line is still at the point \((0, 0.0089)\) as *per* the \(\left(\frac{L}{G}\right)_{\text{min}}\) line.

Since \(Y_1\) **must** still be the **same** (at 0.11) as it is the **feed** (that **cannot** be **changed**), the value of \(X_N\) for the *actual* operating line (with the slope of **144.5**) can be computed as follows

\[
\text{slope} = \frac{\Delta Y}{\Delta X} = \frac{Y_1 - Y_{N+1}}{X_N - X_o}
\]

\[
144.5 = \frac{0.11 - 0.0089}{X_N - 0}
\]

\[
X_N \approx 0.0007
\]
Therefore, the *actual* operating line is the line connecting between

- the point \((X_o, Y_1)\) of \((0, 0.0089)\)

and

- the point \((X_N, Y_{N+1})\) of \((0.0007, 0.11)\)

Step off stages from the point \((X_o, Y_1)\) of \((0, 0.0089)\) to the point \((X_N, Y_{N+1})\) of \((0.0007, 0.11)\) yields the **number** of *equilibrium stages* of \(~3.8\), as shown in the Figure on Page 28
8.3 Stripping Analysis

As we have learned previously, *stripping* is the *operation* that is in the *opposite* direction to the *absorption*.

In the *stripping* operation, the *mixture* of *gas* (i.e. the *solute*) and *liquid* (i.e. the *solvent*) is passed through a *stripping column*, in which the *gaseous* component is to be *stripped off* from the *gas-liquid* mixture by a *stripping gas*, as illustrated in Figure 8.4.
Figure 8.4: A stripping operation
(from “Separation Process Engineering” by Wankat, 2007)

The values of $X_o$ (the inlet concentration of the gas-liquid mixture) and $Y_{N+1}$ (the initial concentration of the stripping gas) are normally given (i.e. known), the value of $X_N$ (the final concentration of the treated liquid stream) is generally specified, and the value of $\frac{L}{G}$ is also given (either directly or indirectly)
Hence, we need to calculate/determine the value of $Y_1$ (the outlet concentration of the stripping gas)

The operating line for stripping is still the same as per the absorption; i.e.

$$Y_{j+1} = \frac{L}{G} X_j + \left( Y_1 - \frac{L}{G} X_o \right)$$

(8.10)

It should be noted, however, that the stripping operation is usually NOT isothermal; hence, its equilibrium line is normally NOT linear

The example of the McCabe-Thiele diagram for the stripping operation is as shown in Figure 8.5
Figure 8.5: A McCabe-Thiele diagram for the stripping operation
(from “Separation Process Engineering” by Wankat, 2007)

Note that the point \((X_N, Y_{N+1})\) and the point \((X_0, Y_1)\) must be on the operating line

As mentioned earlier, the value of \(X_0\) is normally given, as it is the inlet concentration of the feed (the gas-liquid mixture)
To step off stages, we start from the intersection of the operating line and the $X = X_o$ line.

Note that, for stripping, the equilibrium curve is above the operating line; this is because the material (i.e. a gas component in the gas-liquid mixture) migrates from the liquid phase to the gas phase [which is the same as the more volatile component ($MVC$) moves from the liquid phase to the gas phase in the distillation].

Accordingly, for the stripping operation, the maximum amount of stripping gas or the maximum $\frac{L}{G}$ can be determined.
This can be done by drawing a *straight* line from the point of \((X_N, Y_{N+1})\) until it *touches* the *equilibrium curve* at the point where \(X = X_o\).

However, it is *important* to note that this *straight line* (or the *maximum* \(\frac{L}{G}\) line) cannot be *over* (or *higher than*) the *equilibrium line*; thus, *sometimes*, the maximum \(\frac{L}{G}\) line is the *tangential* (but *still* *straight*) line to the *equilibrium line* that originates from the point \((X_N, Y_{N+1})\) to the point where \(X = X_o\), as illustrated as the *dashed lines* in Figure 8.6.
Example We wish to design a *stripping* column to remove carbon dioxide (CO$_2$) from water.

This can be done by heating the water + CO$_2$ mixture and passing it counter-currently with a nitrogen stream in a stripper.

The operation is isothermal and isobaric at 60 °C and 1 atm.

The *carbonated* water contains $9.2 \times 10^{-6}$ mole fraction of CO$_2$ and flows at 100,000 lb$_m$/h.

The *nitrogen* stream enters the column as pure N$_2$ at 1 atm and 60 °C with the volumetric flow rate of 2,500 ft$^3$/h.

Assume that N$_2$ is not dissolved in water and that water is not evaporated.
Given the Henry’s constant for CO$_2$ in water at 60 °C of 3,410 atm/(mole fraction)

If we desire an outlet water concentration of $2.0 \times 10^{-7}$ mole fraction of CO$_2$, find the number of equilibrium stages required

The volume of 1 lb-mol of nitrogen (N$_2$) at 1 atm (14.7 psia) and 60 °C (or 140 °F or 140 + 460 = 600 R) can be calculated, using the equation of state (EoS) for an ideal gas, as follows

$$PV = nRT$$

$$V = \frac{nRT}{P}$$

$$V = \left(1 \text{ lb-mol}\right) \left[10.73 \frac{(\text{ft}^3) (\text{psia})}{(\text{lb-mol})(\text{R})}\right] \left[(140 + 460) \text{ R}\right]$$

$$V = \frac{\left(1 \text{ lb-mol}\right) \left[10.73 \frac{(\text{ft}^3) (\text{psia})}{(\text{lb-mol})(\text{R})}\right] \left[(140 + 460) \text{ R}\right]}{(14.7 \text{ psia})}$$
\[ V = 438 \text{ ft}^3 \]

In other words, we can say that the specific volume \( v \) of \( \text{N}_2 \) is

\[ v = \frac{V}{m} = \frac{438 \text{ ft}^3}{1 \text{ lb-mol}} = 438 \frac{\text{ft}^3}{\text{lb-mol}} \]

It is given that the volumetric flow rate of \( \text{N}_2 \) is 2,500 ft\(^3\)/h, which can be converted to molar flow rate as follows

\[ \frac{2,500 \text{ ft}^3}{h} = \frac{5.71 \text{ lb-mol}}{h} \]

The flow rate of the carbonated water is given as 100,000 lb\(_m\)/h
With the molecular weight of water of 18.02 lb\textsubscript{m}/lb-mol, the molar flow rate of the \textit{carbonated} water can be computed as follows

\[
\frac{100,000 \text{ lb}_m}{\text{h}} \div \frac{18.02 \text{ lb}_m}{\text{lb-mol}} = 5,549 \text{ lb-mol/h}
\]

It is important to note that, since the amount of CO\textsubscript{2} in the \textit{carbonated} water is \textit{extremely low}, it is reasonable to assume that the flow rate of the \textit{carbonated} water (\textit{i.e.} the mixture of water + CO\textsubscript{2}) is about the same as the flow rate of \textit{pure} water (which is found to be 5,549 lb-mol/h)
It is given, in the problem statement, that

- \( x_{in} = x_o = 9.2 \times 10^{-6} = 92 \times 10^{-7} \)
- \( x_{out} = x_N = 2.0 \times 10^{-7} \)
- \( y_{in} = y_{N+1} = 0 \) (pure \( N_2 \))

The schematic diagram for stripping can be presented as shown below
Performing a species balance for CO$_2$:

$$V y_{N+1} + L x_o = V y_1 + L x_N \quad (8.12)$$

gives

$$\begin{align*}
(5.71)(0) + (5549)(9.2 \times 10^{-6}) &= (5.71)y_1 + (5549)(2.0 \times 10^{-7}) \\
y_1 &\approx 0.00875
\end{align*}$$
Note that, as the concentration of the solute is extremely low, the flow rates of the liquid and the gas phases can be assumed to be constant and the $x$ and $y$ co-ordinate can be used.

Accordingly, the **operating line** (dashed lines) passes through two points: \((x_N, y_{N+1})\) and \((x_o, y_1)\) or \((2.0 \times 10^{-7}, 0)\) and \((9.2 \times 10^{-6}, 0.00875)\).

The **equilibrium line** (solid line) can be drawn from the equilibrium equation as follows:

\[
y = \frac{\text{3,410 atm}}{\text{mole fraction}} \times \frac{1 \text{ atm}}{x}
\]

\[
y = 3,410x
\]
By drawing the *operating line* and the *equilibrium line* on the same McCabe-Thiele diagram, we can step off stages, which, in this Question, is found to be \( \sim 3 \) (note that, since this is the *stripping* operation, the *equilibrium* line is *above* the *operating* line).
8.4 Analytical Solution: Kremser Equation

When the concentration of a solute in both gas and liquid phases is very low (< 1%), the total gas and liquid flow rates do not change significantly, even though the solute is transferred from the gas phase to the liquid phase.

Thus, the mole fractions of species $i$ in both gas ($y_i$) and liquid ($x_i$) phases can be used for the calculations (in other words, it is NOT necessary to convert that $y$ and $x$ data to the $Y$ and $X$ coordinate).
For the *dilute* situation described above, Figure 8.2 (on Page 13), which is on *X* and *Y* basis can be replaced by Figure 8.6

![Diagram of a gas absorption](image)

**Figure 8.6:** A gas absorption when the concentration of a solute in both gas and liquid phases are low

(from “Separation Process Engineering” by Wankat, 2007)
For a *dilute* absorber, the operating line is *similar* to that of the *normal* absorption operation (or Eq. 8.10), except that

- $Y$ is replaced by $y$ (*i.e.* the mole fraction of a solute in the *gas* phase)
- $X$ is replaced by $x$ (*i.e.* the mole fraction of a solute in the *liquid* phase)
- $G$ is replaced by $V$

Thus, the operating line for the dilute absorption can be written as follows

$$y_{j+1} = \frac{L}{V} x_j + \left( y_1 - \frac{L}{V} x_o \right) \quad (8.13)$$
All of the assumptions are still the same as per the normal absorption (see Page 7), with an additional assumption that the concentration of the solute in both gas and liquid phases is very low.

To enable the stage-by-stage problem to be solved analytically, an additional assumption must be made, and it is the assumption that the equilibrium line is linear; i.e.

$$y_j = mx_j + b$$  \hspace{1cm} (8.14)

Actually, the Henry’s law equation (Eq. 8.4):

$$y_j = \frac{H_B}{P_{total}} x_j$$

already satisfies this assumption, especially with the assumption that this system is isobaric.
By comparing the Henry’s law equation with Eq. 8.14, it results in the fact that

- \( m = \frac{H_B}{P_{\text{total}}} \)

- \( b = 0 \)

An **analytical** solution for the **absorption** operation can be derived for 2 special cases:

- When the **operating** and **equilibrium** lines are **parallel** to each other (i.e. \( \frac{L}{V} = m \)), as illustrated in Figure 8.7

- When the **operating** and **equilibrium** lines are **NOT parallel** to each other (i.e. \( \frac{L}{V} < m \)), as shown in Figure 8.8
Figure 8.7: The absorption operation for the case that the operating and equilibrium lines are parallel to each other

(from “Separation Process Engineering” by Wankat, 2007)

For the case that the operating and equilibrium lines are parallel to each other or when \( \frac{L}{V} = m \), we obtain the fact, for the absorber with \( N \) equilibrium stages, that

\[
y_{N+1} - y_1 = N \Delta y \tag{8.15}
\]

where
Figure 8.8: The absorption operation for the case that the operating and equilibrium lines are not parallel to each other

(from “Separation Process Engineering” by Wankat, 2007)

\[ \Delta y = y_{j+1} - y_j \]  \hspace{1cm} (8.16)

in which

- \( y_{j+1} \) is obtained from the operating line (Eq. 8.13)
- \( y_j \) is obtained from the equilibrium line (Eq. 8.14)
Combining Eqs. 8.13 & 8.14 with Eq. 8.16 and re-arranging yields

$$\Delta y = \left( \frac{L}{V} - m \right) x_j + \left( y_1 - \frac{L}{V} x_o - b \right)$$

(8.17)

Since, in this case, \( \frac{L}{V} = m \), Eq. 8.17 becomes

$$\Delta y = \left( y_1 - \frac{L}{V} x_o - b \right) = \text{constant}$$

(8.18)

Note that Eq. 8.18 is true because the operating and equilibrium lines are parallel to each other; thus, the distance between the operating and the equilibrium lines or \( \Delta y \) are \textit{constant}. 52
Substituting Eq. 8.18:

\[ \Delta y = \left( y_1 - \frac{L}{V} x_o - b \right) \]  

(8.18)

into Eq. 8.15:

\[ y_{N+1} - y_1 = N \Delta y \]  

(8.15)

and re-arranging the resulting equation yields

\[ N \left( y_1 - \frac{L}{V} x_o - b \right) = y_{N+1} - y_1 \]

\[ N = \frac{y_{N+1} - y_1}{y_1 - \frac{L}{V} x_o - b} \]  

(8.19)

**Eq. 8.18** is a special case of the Kremser equation, when \( \frac{L}{V} = m \) \( \left( \text{or} \quad \frac{L}{mV} = 1 \right) \)
For the case where $\frac{L}{V} < m$ (see Figure 8.8 on Page 51), $\Delta y$ is no longer constant or $\Delta y$ varies from stage to stage.

Re-arranging Eq. 8.14:

$$y_j = mx_j + b \quad (8.14)$$

results in

$$x_j = \frac{y_j - b}{m} \quad (8.20)$$

Substituting Eq. 8.20 into Eq. 8.17:

$$\Delta y = \left( \frac{L}{V} - m \right) x_j + \left( y_1 - \frac{L}{V} x_o - b \right) \quad (8.17)$$

and re-arranging for $\Delta y$ of the stages $j$ and $j + 1$ yields
\[(\Delta y)_{j} = \left(\frac{L}{mV} - 1\right)y_{j} + \left(y_{1} - \frac{L}{mV}b - \frac{L}{V}x_{o}\right)\]  \hspace{1cm} (8.21)

and

\[(\Delta y)_{j+1} = \left(\frac{L}{mV} - 1\right)y_{j+1} + \left(y_{1} - \frac{L}{mV}b - \frac{L}{V}x_{o}\right)\]  \hspace{1cm} (8.22)

\[(8.22) - (8.21) \text{ and re-arranging gives}\]

\[\left(\Delta y\right)_{j+1} = \frac{L}{mV}\left(\Delta y\right)_{j}\]  \hspace{1cm} (8.23)

Eq. 8.23 provides the relationship of $\Delta y$ for the adjacent stages (stage ที่ติดกัน; e.g., stage 1 and stage 2) with the coefficient of $\frac{L}{mV}$ (commonly called the “absorption factor”)

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Since $\Delta y$ is NOT constant, Eq. 8.15 is written as

$$y_{N+1} - y_1 = \Delta y_1 + \Delta y_2 + \Delta y_3 \ldots + \Delta y_N$$

(8.24)

By employing Eq. 8.23, we obtain the fact that, e.g.,

$$\left(\Delta y\right)_2 = \frac{L}{mV}\left(\Delta y\right)_1$$

(8.25)

$$\left(\Delta y\right)_3 = \frac{L}{mV}\left(\Delta y\right)_2$$

(8.26)

Substituting Eq. 8.25 into Eq. 8.26 yields

$$\left(\Delta y\right)_3 = \frac{L}{mV}\left[\frac{L}{mV}\left(\Delta y\right)_1\right]$$

$$\left(\Delta y\right)_3 = \left(\frac{L}{mV}\right)^2\left(\Delta y\right)_1$$

(8.27)
Eq. 8.27 can be written in a general form as

\[
\left( \Delta y \right)_{j+1} = \left( \frac{L}{mV} \right)^j \left( \Delta y \right)_1 \tag{8.28}
\]

By applying Eq. 8.28 with Eq. 8.24:

\[y_{N+1} - y_1 = \Delta y_1 + \Delta y_2 + \Delta y_3 \ldots + \Delta y_N\tag{8.24}\]

we obtain the following equation:

\[y_{N+1} - y_1 = \left[ 1 + \left( \frac{L}{mV} \right) + \left( \frac{L}{mV} \right)^2 + \ldots + \left( \frac{L}{mV} \right)^{N-1} \right] \Delta y_1 \tag{8.29}\]

After performing mathematical manipulation for the summation on the right hand side (RHS) of Eq. 8.29 and re-arranging the resulting equation, it results in
\[ y_{N+1} - y_1 = \frac{1 - \left( \frac{L}{mV} \right)^N}{1 - \left( \frac{L}{mV} \right)} \Delta y_1 \]

or

\[ \frac{y_{N+1} - y_1}{\Delta y_1} = \frac{1 - \left( \frac{L}{mV} \right)^N}{1 - \left( \frac{L}{mV} \right)} \quad (8.30) \]

Writing Eq. 8.23:

\[ (\Delta y)_{j+1} = \frac{L}{mV} (\Delta y)_j \quad (8.23) \]

for stage 1 gives

\[ (\Delta y)_{o+1} = \frac{L}{mV} (\Delta y)_o \]

or
\[(\Delta y)_1 = \frac{L}{mV}(\Delta y)_o\] \hspace{1cm} (8.31)

where \((\Delta y)_o = \Delta y \text{ (at } x = x_o) = y_1 - y_1^*\) (see Figure 8.8 on Page 51)

Note that, from the equilibrium line/equation,
\[y_1^* = mx_o + b\] \hspace{1cm} (8.32)

Thus, Eq. 8.31 can be re-written as
\[\left(\Delta y\right)_1 = \frac{L}{mV}(y_1 - y_1^*)\] \hspace{1cm} (8.33)

Substituting Eq. 8.33 into Eq. 8.30 and re-arranging gives
\[
\frac{y_{N+1} - y_1}{\frac{L}{mV}(y_1 - y_1^*)} = 1 - \left(\frac{L}{mV}\right)^N
\]
\[ \frac{y_{N+1} - y_1}{(y_1 - y_1^*)} = \frac{L}{mV} - \left( \frac{L}{mV} \right)^{N+1} \] \tag{8.34}

Solving Eq. 8.34 for \( N \) yields

\[
N = \frac{\ln \left( \left( 1 - \frac{mV}{L} \right) \frac{y_{N+1} - y_1^*}{y_1 - y_1^*} \right) + \frac{mV}{L}}{\ln \left( \frac{mV}{L} \right)}
\]

\tag{8.35}

(note that, in this case, \( \frac{L}{V} \neq m \) or \( \frac{L}{mV} \neq 1 \))

Eq. 8.35 is another form of the Kremser equation, when \( \frac{L}{mV} \neq 1 \)
The Kremser equations in terms of gas-phase composition can be written as follows

\[ \frac{y_{N+1} - y_1}{y_1 - y_1^*} = \frac{\left( \frac{L}{mV} \right) - \left( \frac{L}{mV} \right)^{N+1}}{1 - \left( \frac{L}{mV} \right)^{N+1}} \]  
(8.34)

\[ \frac{y_{N+1} - y_1^*}{y_1 - y_1^*} = \left( \frac{L}{mV} \right)^N \]  
(8.36)

\[ \ln \left( \frac{\left( y_{N+1} - y_1^* \right)}{\left( y_1 - y_1^* \right)} \right) \]  
(8.37)

\[ \ln \left( \frac{\left( y_{N+1} - y_1^* \right)}{\left( y_1 - y_1^* \right)} \right) \]  
(8.38)

where \( y_{N+1}^* = mx_N + b \) and \( y_1^* = mx_0 + b \)
The Kremser equations in terms of liquid-phase composition are

\[ N = \frac{\ln \left[ \left( 1 - \frac{L}{mV} \right) \left( \frac{x_o - x_N^*}{x_N - x_N^*} \right) + \frac{L}{mV} \right]}{\ln \left( \frac{mV}{L} \right)} \]  
(8.39)

\[ N = \frac{\ln \left[ \left( \frac{x_N - x_N^*}{x_o - x_o^*} \right) \left( \frac{x_o - x_N^*}{x_o - x_N} \right) \left( \frac{x_o - x_N^*}{x_o - x_N} \right) \right]}{\ln \left[ \left( \frac{x_o - x_N^*}{x_o - x_N} \right) \left( \frac{x_o - x_N^*}{x_o - x_N} \right) \right]} \]  
(8.40)

\[ \frac{x_N - x_N^*}{x_o - x_N^*} = \frac{1 - \left( \frac{mV}{L} \right)}{1 - \left( \frac{mV}{L} \right)^{N+1}} \]  
(8.41)
\[ \frac{x_N^* - x_N}{x_o^* - x_o} = \left( \frac{L}{mV} \right)^N \] (8.42)

where \( x_N^* = \frac{y_{N+1} - b}{m} \) and \( x_o^* = \frac{y_1 - b}{m} \)

Among many equations above, how to choose the appropriate equation(s) depends on the data given in the problem statement.

Let’s examine the following Example, which illustrates how to select the proper Kremser equation for a specified problem
Example  A plate tower with 6 equilibrium stages is employed for *stripping* ammonia from waste water with the *inlet* concentration of 0.10 mol% using counter-current air at atmospheric pressure and 80 °F

Determine the concentration of ammonia in the *exit* waste water if the *stripping* air is *ammonia-free* and the feed rate of air is 30 standard cubic feet (scf) *per* 1 lb_m of waste water

Given the equilibrium equation of ammonia at 80 °F as \( y = 1.414x \)

One (1) lb-mol of air is equivalent to the *volume* \((\bar{V})\) of 379 standard cubic feet (scf) [for 60 °F (519.67 R) and 14.7 psi (lb_f/in^2)], which can be computed as follows
\[
\bar{V} = \frac{nRT}{P} (1 \text{ lb-mol}) \left[ 10.73 \left( \frac{\text{psia}}{\text{lb-mol}} \right) \left( \frac{\text{ft}^3}{\text{R}} \right) \left[ (60 + 459.67) \text{ R} \right] \right] \div (14.7 \text{ psia}) \\
\bar{V} = 379 \text{ ft}^3 \left( \text{scf} \right)
\]

Thus, the molar flow rate of the stripping gas \( V \) is

\[
V = \frac{30 \text{ scf air}}{379 \text{ scf air} \div \text{lb-mol air}} = 0.0792 \text{ lb-mol air}
\]

The molecular weight (MW) of water is 18.02 \( \frac{\text{lb}}{\text{lb-mol}} \); hence, the molar flow rate of the waste water \( L \) can be calculated as follows
\[ L = \frac{1 \text{ lb}_m \text{ water}}{18.02 \frac{\text{lb}_m \text{ water}}{\text{lb-mol water}}} = 0.0555 \text{ lb-mol water} \]

The given data are (see the Figure below)

\[ x_0 = 0.001 \]

\[ y_1 \]

\[ x_6 \]

\[ y_7 = 0.30 \text{ ft}^3 \text{(std.) air/lb water} \]

\[ p = 1 \text{ atm} \]

\[ 80^\circ \text{F} \]

(from “Separation Process Engineering” by Wankat, 2007)

- The inlet concentration of ammonia (NH\(_3\)) in the waste water: \( x_o = \frac{0.1}{100} = 0.001 \)
• The inlet concentration of NH$_3$ in the stripping gas or air: $y_{N+1} = y_7 = 0$ (as the air is ammonia-free)

• $m = 1.414$ and $b = 0$ [from the given equilibrium equation: $y = 1.414x$, compared to the standard equilibrium equation (Eq. 8.14): $y = mx + b$]

• The number of equilibrium stages $(N) = 6$

Thus, by employing Eq. 8.41:

$$\frac{x_N - x_N^*}{x_o - x_N^*} = \frac{1 - \left(\frac{mV}{L}\right)}{1 - \left(\frac{mV}{L}\right)^{N+1}}$$  \hspace{1cm} (8.41)

with the fact that

$$x_N^* = \frac{y_{N+1} - b}{m}$$
which yields

\[ x_N^* = \frac{0 - 0}{1.414} = 0 \]

we can compute the value of the **outlet concentration** of \( \text{NH}_3 \) in the **waste water** \((x_N \text{ or } x_6)\) as follows

\[
x_N - x_N^* = \left[ 1 - \frac{mV}{L} \right] \left[ 1 - \left( \frac{mV}{L} \right)^{N+1} \right] \left( x_o - x_N^* \right)
\]

\[
x_N - 0 = \left[ 1 - \frac{(1.414)(0.0792)}{(0.0555)} \right] \left[ 1 - \left[ \frac{(1.414)(0.0792)}{(0.0555)} \right]^{6+1} \right] (0.0010 - 0)
\]

\[
x_N = 7.45 \times 10^{-6} = 7.45 \text{ ppm}
\]