# Chapter 8:

# Absorption and Stripping

In addition to the distillation, there are other unit operations used for separating substances

**Absorption** is the **unit operation** in which one or more components of a **gas** stream are **removed** from the *gas mixture* by being **absorbed** onto a *non-volatile* **liquid** (called a **"solvent"**)

In this case, the **solvent** is the ***separating*** **agent**

**Stripping** is the operation that is in the opposite direction to the **absorption**, in which one or more **gaseous** components in a **liquid** stream is **removed** from the *gas-liquid* solution by being **vaporised** into an *insoluble* **gas** stream

In the **stripping** operation, the *insoluble* **gas stream** is the ***separating*** **agent**

**What is the *separating* agent**

**in the *distillation*?**

**Absorption** can be

- ***physical***, when the *solute* is *dissolved* into the *solvent* because it has *higher solubility* in the solvent than other gases
- ***chemical***, when the *solute reacts with the solvent*, and the resulting product still remains in the solvent

  Normally, a *reversible* reaction between the solute and the solvent is preferred, in order for the solvent to be regenerated (นำกลับมาใช้ใหม่)

Similar to the *distillation*, both *absorption* and *stripping* are operated as *equilibrium* stage operations, in which *liquid* and *vapour* are *in contact* with each other

However, in both absorption and stripping operations, the columns are simpler than the distillation column; normally, they do **not** *need condensers* and *re-boilers* as *per* the distillation

An *example* of the uses of *absorption* and *stripping* is a **gas treatment plant**, shown in Figure 8.1

The **mixture** of the **gases** (*i.e.* the gas to be treated in Figure 8.1), from which one component (*e.g.*, $CO_2$ or $H_2S$) in the gas mixture needs to be removed, is passed through an **absorber**, in which a *liquid-phase* solvent (*e.g.*, MEA or amine solvent in water) is passed though, to absorb that gaseous component

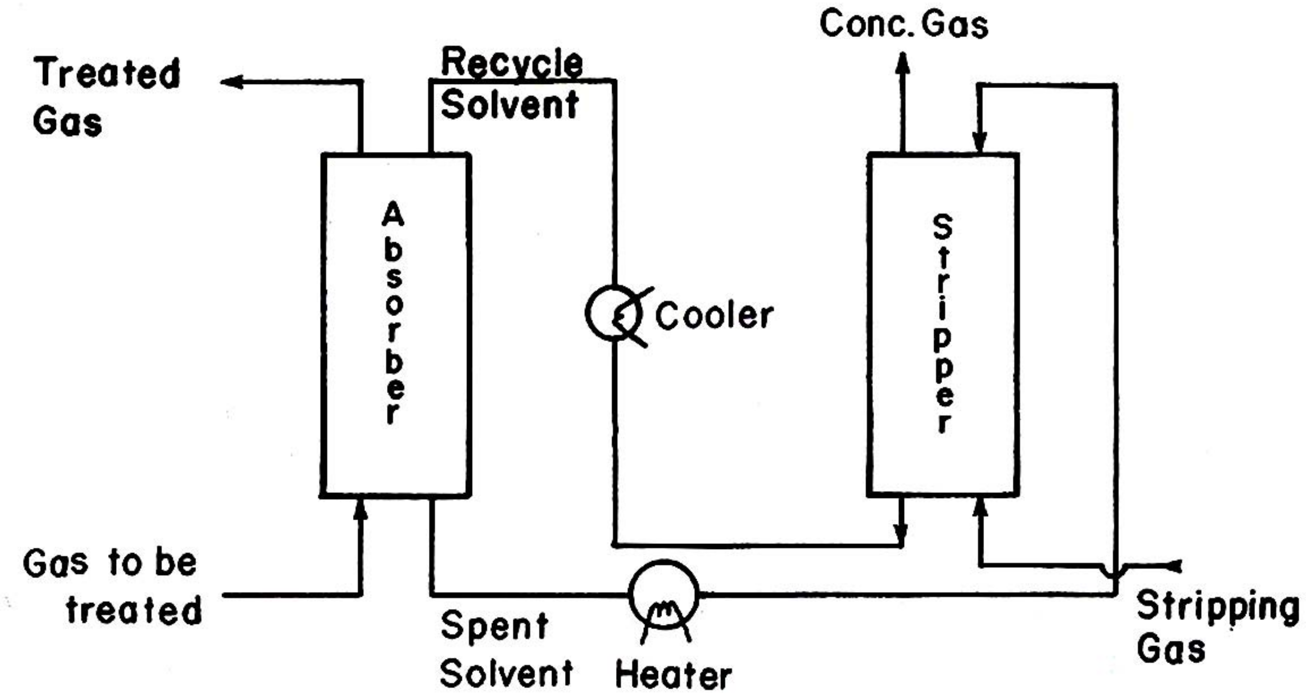


**Figure 8.1: A gas treatment plant**

(from “Separation Process Engineering” by Wankat, 2007)

The *removal* component is *mixed* (physically or chemically) with the *solvent*

The *resulting* solvent is heated by a heater and becomes the *saturated* solvent (so that the resulting solvent can be stripped)

The resulting *saturated* solvent is, subsequently, passed through a *stripper* (or a *stripping column*), in which the **gaseous** component in the *saturated* solvent is **stripped off** by a *stripping gas* (*e.g.*, steam)

In this Example (Figure 8.1), the *desired product*

- for an absorber is a *treated gas* stream
- for a stripper is a *recycle solvent* stream

## 8.1 Absorption and Stripping Equilibria

**Absorption** and **stripping** *involves*, at least, **3 components** (as illustrated recently) and **2 phases**

Generally, for *simplicity*, we often *assume* the following:

- *Carrier gas* is *insoluble* (into a solvent)
- *Solvent* is *non-volatile* (thus, the *loss* of solvent due to *vaporisation* is *negligible*)
- The *system* is *isothermal* (constant temperature) and *isobaric* (constant pressure)

Since the *absorption* or *stripping* system comprises **3 species** and **2 phases**, the degree of freedom $\left(F\right)$, calculated using the Gibbs phase rule, is

$$\boldsymbol{F} = C - P + 2 = 3 - 2 + 2 = \mathbf{3}$$

As we assume that the system is

- *isothermal*, which results in the fact that the system's **temperature** is **constant**
- *isobaric*, which results in the fact that the system's **pressure** is **constant**

the degree of freedom $\left(F\right)$ is reduced to one (1)

Normally, an **equilibrium data** is used as the *remaining* degree of freedom $\left(F\right)$

In general, the **amount** of **solute** (*i.e.* a gas to be removed from the gas mixture) is relatively **low**

For a *low* concentration of a *solute*, a **Henry's law** is employed to express the **equilibrium** between the *concentration* (*e.g.*, mole fraction or percent) of the *solute* in the *gas* phase and that in the *liquid* phase as follows

$$P_B = H_B x_B \tag{8.1}$$

where

$P_B$ = partial pressure of the **solute B** in the *gas* phase (or in the **gas mixture**)

$x_B$ = concentration (in *mole fraction*) of the **solute B** in the *liquid* phase

$H_B$ = Henry's constant for the **solute B**

A **mole fraction** of the **solute B** in the *gas* phase $\left(y_B\right)$ can be described by the following equation:

$$y_B = \frac{P_B}{P_{\text{total}}} \tag{8.2}$$

which can be re-arranged to

$$P_B = y_B P_{\text{total}} \tag{8.3}$$

Combining Eq. 8.3 with Eq. 8.1 and re-arranging the resulting equation gives

$$y_B = \frac{H_B}{P_{\text{total}}} x_B \tag{8.4}$$

As we have assumed previously that the system is *isobaric* [which results in the fact the system's *pressure* $\left(P_{total}\right)$ is *constant*], Eq. 8.4, which is an *equilibrium* equation for *absorption* and *stripping* operations, is a **linear** relationship with the **slope** of $\boldsymbol{\frac{H_B}{P_{total}}}$

## 8.2 Operating Lines for Absorption

We have learned previously from the distillation operation that if the **operating line** is **linear**, it would be *easy to use*

To have a *linear* or *straight* operating line, it is required that

- the *energy balance* is ***automatically*** **satisfied**
- both **liquid** and **gas** (or **vapour**) **flow rates** are ***constant***

In order for the *energy balance* to be *automatically satisfied*, we have to assume that

- the **heat of absorption** is ***negligible***
- the **operation** is ***isothermal***
- a **solvent** is ***non-volatile***
- a **carrier gas** is ***insoluble*** (into the solvent)

For any **gas absorber**, as illustrated in Figure 8.2, by employing the above assumptions, we obtain the facts that

$$L_1 = L_2 = ... = L_N = L \tag{8.5}$$

$$G_N = G_{N-1} = ... = G_1 = G \tag{8.6}$$

It is important to note that, *to make* the *liquid* and *gas* flow rates **constant**, especially for the *concentrated* mixture (of either *liquid* or *gas* phase), we **cannot use** the ***overall*** or ***total*** **flow rates** of ***gas*** and ***liquid***

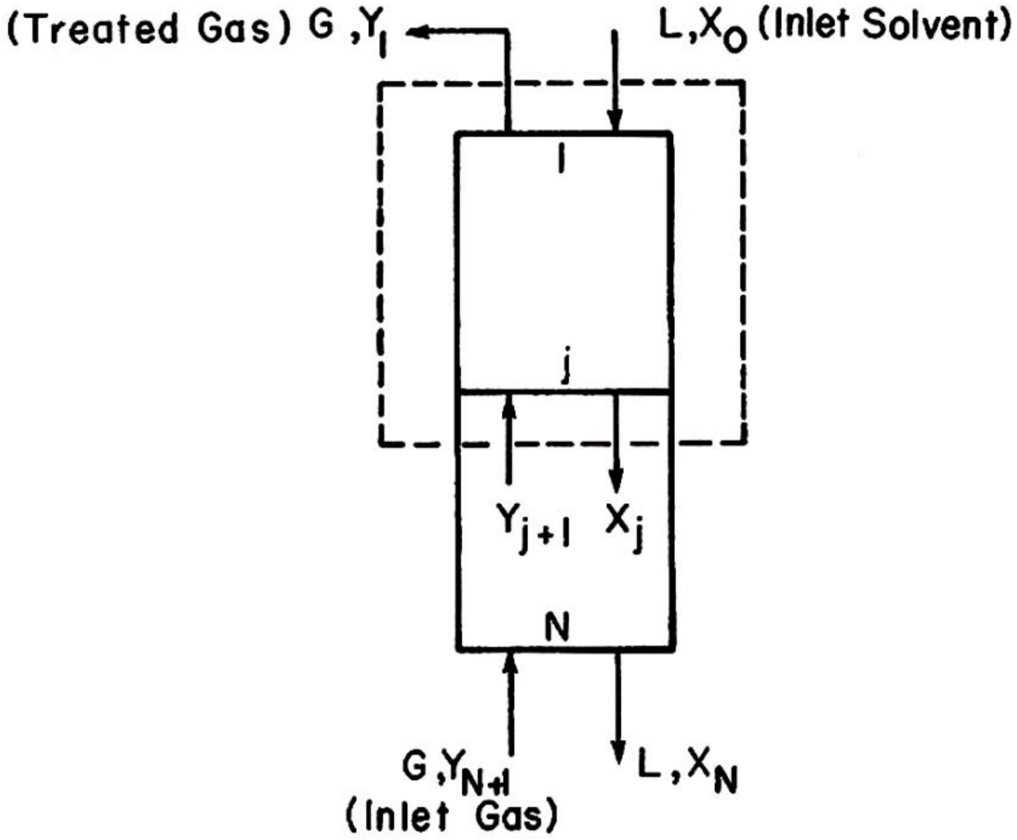


**Figure 8.2: A gas absorption column**

(from "Separation Process Engineering" by Wankat, 2007)

In other words,

- the **liquid** flow rate $\left(L\right)$ must be the *molar* or *mass* flow rate of a *non-volatile* **solvent**
- the **gas** flow rate $\left(G\right)$ must be the *molar* or *mass* flow rate of an *insoluble* **carrier gas**

Accordingly, the *mole fractions* of solute B in the gas $\left(y_B\right)$ and the liquid $\left(x_B\right)$ phases, which are defined as

- $$y_B = \frac{\text{moles of solute B (in the gas phase)}}{\text{total moles of a gas mixture}}$$

or

$$y_B = \frac{\text{moles of solute B (in the gas phase)}}{\begin{bmatrix}\text{moles of carrier gas A} \\ + \text{ moles of solute B}\end{bmatrix}}$$

and

- $$x_B = \frac{\text{moles of solute B (in the liquid phase)}}{\text{total moles of a solution}}$$

or

$$x_B = \frac{\text{moles of solute B (in the liquid phase)}}{\begin{bmatrix}\text{moles of a solvent} \\ + \text{ moles of solute B}\end{bmatrix}}$$

have to be modified to

- $Y_B = \dfrac{\text{moles of solute B}}{\text{moles of } \textit{pure} \text{ carrier gas A}}$
- $X_B = \dfrac{\text{moles of solute B}}{\text{moles of } \textit{pure} \text{ solvent}}$

The relationships between $Y_B$ and $y_B$ and between $X_B$ and $x_B$ can be written as follows

$$Y_B = \frac{y_B}{1 - y_B} \tag{8.7}$$

$$X_B = \frac{x_B}{1 - x_B} \tag{8.8}$$

Thus, by performing *material balances* for the *given envelope* in *Figure 8.2*, we obtain the following equation:

$$GY_{j+1} + LX_o = GY_1 + LX_j \tag{8.9}$$

Re-arranging Eq. 8.9 for $Y_{j+1}$ results in

$$Y_{j+1} = \frac{L}{G} X_j + \left( Y_1 - \frac{L}{G} X_o \right) \tag{8.10}$$

Eq. 8.10 is an *operating line* for **absorption**, with

- the slope of $\frac{L}{G}$
- the Y-intercept of $Y_1 - \frac{L}{G} X_o$

In order to obtain the **number** of **equilibrium stages** required for the **absorption** operation from the initial concentration of $X_o$ to the final concentration of $X_N$, the following **procedure**, which is ***similar*** to that for the **distillation** operation, is employed

1) Draw an *equilibrium* line on the $X$-$Y$ co-ordinate

2) Draw an *operating* line; the values of $X_o$, $Y_{N+1}$, $Y_1$, and $\frac{L}{G}$ are generally known – note that the point $\left(X_o, Y_1\right)$ must be on the operating line

3) Locate the point $\left(X_o, Y_1\right)$ and step off stages between the *operating* line and the *equilibrium* line from X = $X_o$ until it reaches the final concentration of $X_N$

This $X$-$Y$ diagram, which consists of the *operating* and *equilibrium* lines for the **absorption** operation, is still called the "**McCabe-Thiele** diagram"

An example of the *McCabe-Thiele* diagram for *absorption* is illustrated in Figure 8.3

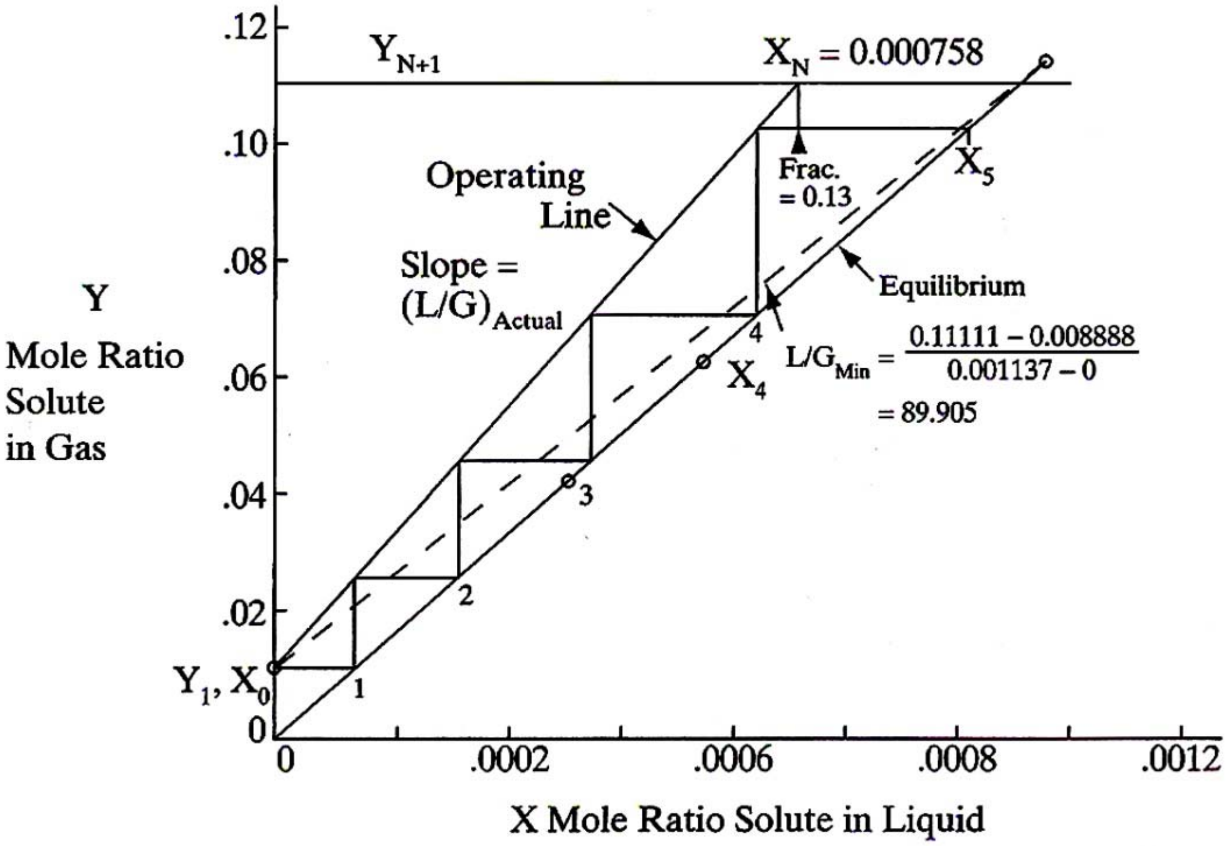


**Figure 8.3: A McCabe-Thiele diagram for the absorption operation**

(from "Separation Process Engineering" by Wankat, 2007)

Note that the *operating* line in the McCabe-Thiele diagram for **absorption** is **above** the *equilibrium* line

This is **because** the **solute** is **transferred from** the ***gas*** **phase** (*i.e.* the gas mixture) **to** the ***liquid*** **phase** (*i.e.* the solvent), which is in the **opposite direction** to the **distillation** operation, in which the material (or the *more volatile component*: MVC) is transferred from the *liquid* phase to the *gas* phase

The dotted lines in Figure 8.3 is the *minimum* $\frac{L}{G}$ line or the $\left(\frac{L}{G}\right)_{\min}$ line; it is the line drawing from the initial point of $\left(X_o,\ Y_1\right)$ until *touching* the *equilibrium* line at the point where $\mathrm{Y} = Y_{N+1}$ (note that $Y_{N+1}$ is the concentration of the *solute* in the ***inlet*** *gas stream*)

It is important to note that, if the **system** is **NOT isothermal** (as in the case of the *distillation* operation), the *operating* line is still *linear*, but the **equilibrium** line is ***no longer*** **linear**

The following Example illustrates how to calculate the required **number** of **equilibrium stages** for the **absorption** operation

**Example** A gas stream containing 90 mol% $N_2$ and 10% $CO_2$ is passed through an absorber, in which *pure* and cool water at 5 °C is used as a solvent

The operation is assumed to be isothermal at 5 °C and isobaric at 10 atm

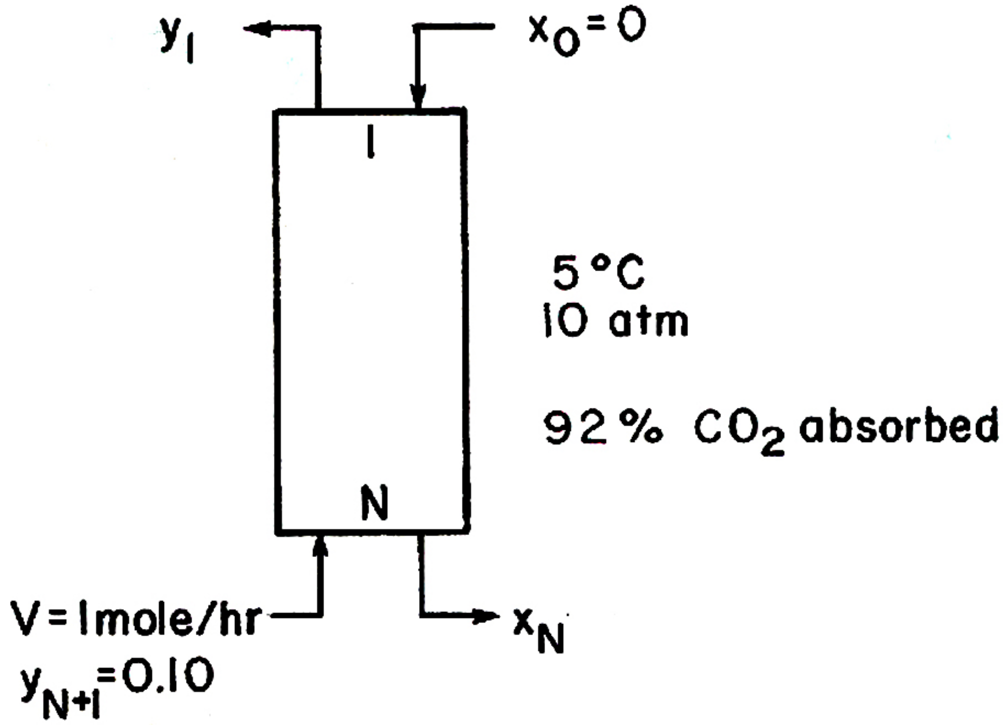
The liquid flow rate is 1.5 times the minimum liquid flow rate $\left(\frac{L}{G}\right)_{\min}$

Determine the number of equilibrium stages required to absorb 92 mol% of $CO_2$

Given Henry's constant of $CO_2$ in water at 5 °C of 876 atm/mole fraction

**Basis:** 100 mol/h of the gas mixture

The schematic diagram for this absorption operation can be illustrated as follows



(from "Separation Process Engineering" by Wankat, 2007)

Substituting given numerical values into Eq. 8.4:

$$y = \frac{H}{P_{\text{total}}} x \tag{8.4}$$

gives

$$y = \frac{876}{10} x = 87.6x \tag{8.11}$$

Eq. 8.11 is the equilibrium equation; thus, the equilibrium or the $y$-$x$ data can be computed and summarised in the following Table:

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 0.0001 | 0.00876 |
| 0.0004 | 0.0350 |
| 0.0006 | 0.0526 |
| 0.0008 | 0.0701 |
| 0.0010 | 0.0876 |
| 0.0012 | 0.1051 |

However, both $x$ and $y$ values have to be converted to $X$ and $Y$ as exemplified on the next Page

- For $x = 0.0001$:

$$X = \frac{x}{1-x} = \frac{0.0001}{1-0.0001} \approx 0.0001$$

- For $y = 0.00876$:

$$Y = \frac{y}{1-y} = \frac{0.00876}{1-0.00876} \approx 0.00884$$

Thus, the *equilibrium* data on $Y$-$X$ co-ordinate are as follows

| $\boldsymbol{X}$ | $\boldsymbol{Y}$ |
|---|---|
| 0 | 0 |
| 0.0001 | 0.00884 |
| 0.0004 | 0.0363 |
| 0.0006 | 0.0555 |
| 0.0008 | 0.0754 |
| 0.0010 | 0.0960 |
| 0.0012 | 0.1175 |

Since the basis of calculation is 100 mol/h of the gas mixture, the amount in "mole" of $CO_2$ in the *input* stream is

$$\left(n_{\mathrm{CO_2}}\right)_{\mathrm{inlet}} = \left(\frac{10}{100}\right) \times 100 \text{ mol/h} = 10 \text{ mol/h}$$

The amount of carrier gas (*i.e.* $N_2$ in this Example) is given as 90%, meaning that the **flow rate** of the **carrier gas** (**only**) is

$$n_{\mathrm{N_2}} = \left(\frac{90}{100}\right) \times 100 \text{ mol/h} = 90 \text{ mol/h}$$

From the problem statement, it is required that 92 mol% of $CO_2$ is absorbed by the solvent (*i.e.* cool water); this means that only 8% of $CO_2$ flowing into the absorber remains in the gas mixture

Accordingly, the amount in “mole” of $CO_2$ in the *outlet* gas stream can be calculated as follows

$$\left(n_{\mathrm{CO_2}}\right)_{\mathrm{outlet}} = \left(\frac{8}{100}\right) \times \left(n_{\mathrm{CO_2}}\right)_{\mathrm{inlet}}$$

$$\left(n_{\mathrm{CO_2}}\right)_{\mathrm{outlet}} = \left(\frac{8}{100}\right) \times 10 \text{ mol/h} = 0.8 \text{ mol/h}$$

Hence, the *inlet* concentration (in “mole ratio”) of $CO_2$ for the gas phase $\left(Y_{N+1}\right)$ is

$$Y_{N+1} = \left(\frac{n_{\mathrm{CO_2}}}{n_{\mathrm{N_2}}}\right)_{\mathrm{inlet}} = \frac{10}{90} = 0.11$$

and the *outlet* concentration (in “mole ratio”) of the $CO_2$ in the gas phase $\left(Y_1\right)$ is

$$Y_1 = \left(\frac{n_{\mathrm{CO_2}}}{n_{\mathrm{N_2}}}\right)_{\mathrm{outlet}} = \frac{0.8}{90} = 0.0089$$

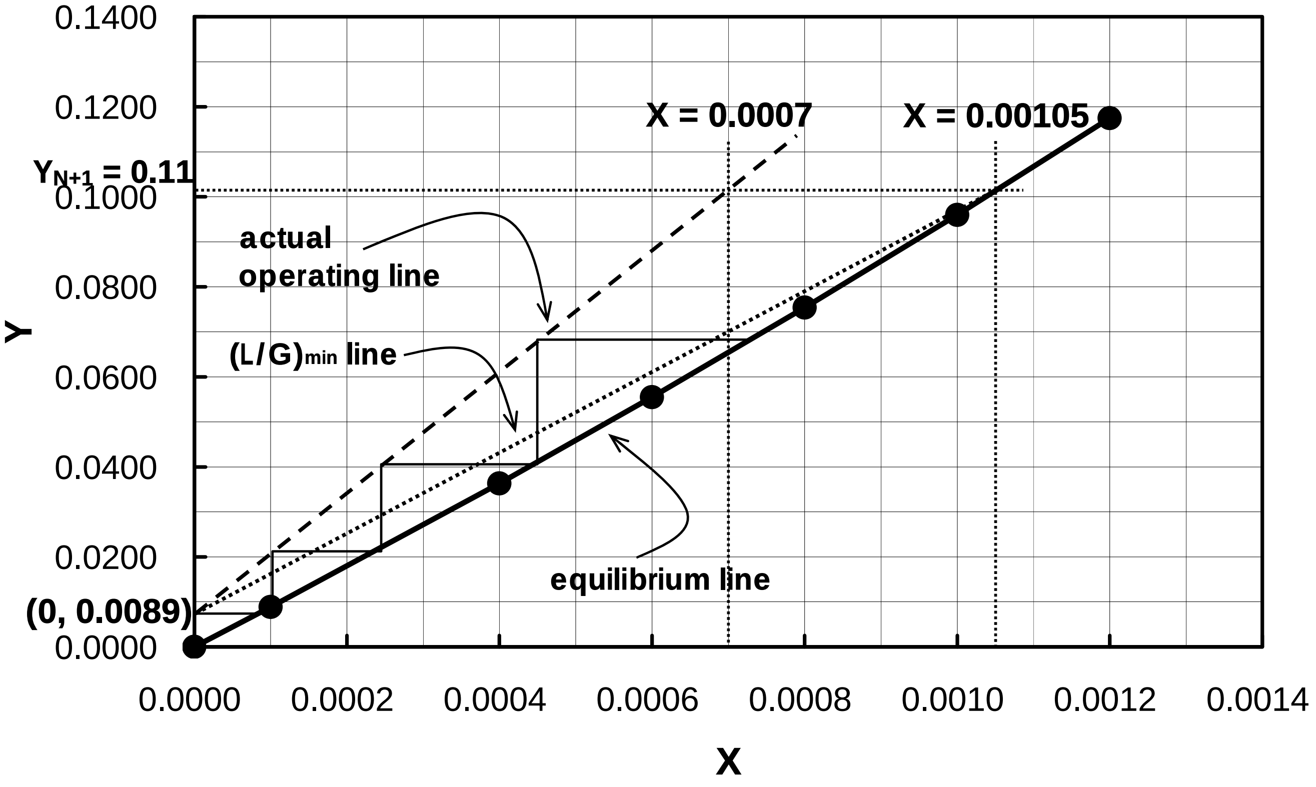
Since *pure* water is used as the solvent, it results in the fact that $x_o = 0$, which means that

$$X_o = 0$$

Accordingly, the origin of the operating line is the point $\left(X_o, Y_1\right)$ of $\left(0, 0.0089\right)$

The *equilibrium* line (from the equilibrium data on Page 24) can be plotted on the ***Y*-*X*** coordinate as shown on Page 28 (as a *solid* line)

The $\left(\frac{L}{G}\right)_{\min}$ line is the line originates from the point of $\left(0, 0.0089\right)$ and **touches** the equilibrium line at $\mathbf{Y} = \boldsymbol{Y_{N+1}} = \mathbf{0.11}$; note that the **X value** at the **touching** point can be read as **0.00105**



The **slope** of the $\left(\frac{L}{G}\right)_{\min}$ line (the dotted lines) is found to be **96.3** $\left[\frac{\Delta Y}{\Delta X} = \frac{(0.11 - 0.0089)}{(0.00105 - 0)}\right]$

Thus, the *actual* **slope** of the **operating** line is

$$1.5 \times \left(\frac{L}{G}\right)_{\min} = 1.5 \times 96.3 = \mathbf{144.5}$$

The origin of the *actual* operating line is still at the point $\left(0, 0.0089\right)$ as *per* the $\left(\dfrac{L}{G}\right)_{\min}$ line

Since $\boldsymbol{Y_1}$ **must** still **be** the **same** (at 0.11) as it is the **feed** (that *cannot* be *changed*), the value of $X_N$ for the *actual* operating line (with the **slope** of **144.5**) can be computed as follows

$$\text{slope} = \frac{\Delta Y}{\Delta X} = \frac{Y_1 - Y_{N+1}}{X_N - X_o}$$

$$144.5 = \frac{\left(0.11 - 0.0089\right)}{\left(X_N - 0\right)}$$

$$X_N \approx 0.0007$$

Therefore, the *actual* operating line is the line connecting between

- the point $\left(X_o, Y_1\right)$ of $\left(0, 0.0089\right)$

and

- the point $\left(X_N, Y_{N+1}\right)$ of $\left(0.0007, 0.11\right)$

Step off stages from the point $\left(X_o, Y_1\right)$ of $\left(0, 0.0089\right)$ to the point $\left(X_N, Y_{N+1}\right)$ of $\left(0.0007, 0.11\right)$ yields the **number** of *equilibrium* **stages** of **~3.8**, as shown in the Figure on Page 28

## 8.3 Stripping Analysis

As we have learned previously, *stripping* is the *operation* that is in the *opposite* direction to the *absorption*

In the *stripping* operation, the **mixture** of **gas** (*i.e.* the *solute*) **and liquid** (*i.e.* the *solvent*) is passed through a **stripping column**, in which the **gaseous** component is to be **stripped off** from the *gas-liquid* mixture by a **stripping *gas***, as illustrated in Figure 8.4

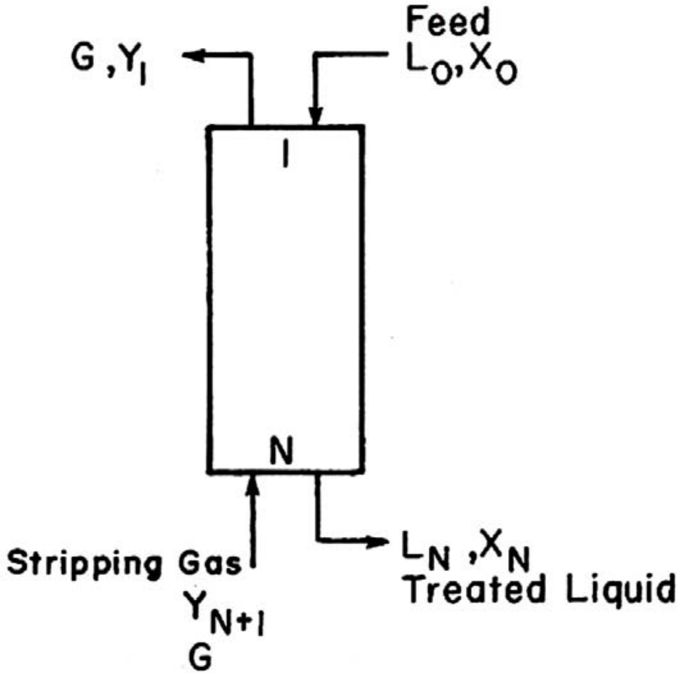


**Figure 8.4: A stripping operation**

(from “Separation Process Engineering” by Wankat, 2007)

The values of $X_o$ (the inlet *concentration* of the *gas-liquid mixture*) and $Y_{N+1}$ (the *initial concentration* of the *stripping gas*) are normally given (*i.e.* known), the value of $X_N$ (the *final concentration* of the *treated liquid stream*) is generally specified, and the value of $\frac{L}{G}$ is also given (either directly or indirectly)

Hence, we need to calculate/determine the value of $Y_1$ (the *outlet concentration* of the *stripping gas*)

The **operating line** for **stripping** is still the **same** as *per* the *absorption*; *i.e.*

$$Y_{j+1} = \frac{L}{G} X_j + \left( Y_1 - \frac{L}{G} X_o \right) \tag{8.10}$$

It should be noted, however, that the *stripping* operation is usually **NOT** *isothermal*; hence, its **equilibrium line** is normally **NOT** ***linear***

The example of the McCabe-Thiele diagram for the *stripping* operation is as shown in Figure 8.5

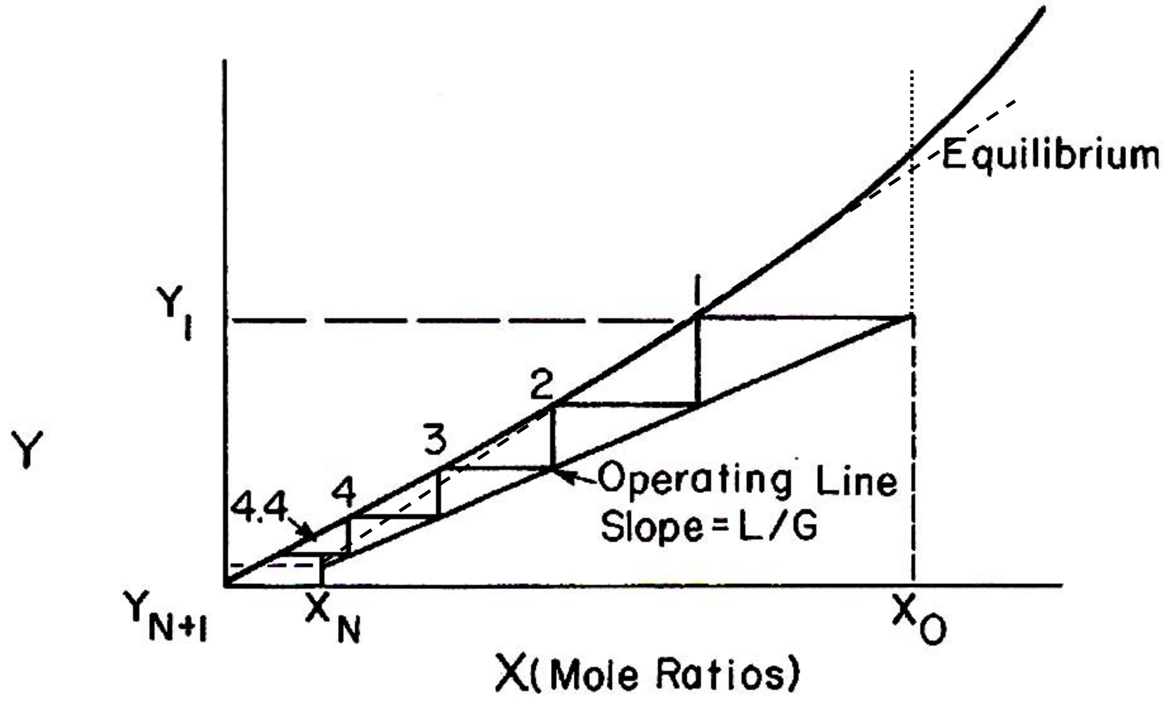


**Figure 8.5: A McCabe-Thiele diagram for the stripping operation**

(from "Separation Process Engineering" by Wankat, 2007)

Note that the point $\left(X_N, Y_{N+1}\right)$ and the point $\left(X_o, Y_1\right)$ must be on the **operating line**

As mentioned earlier, the value of $X_o$ is normally given, as it is the *inlet concentration* of the *feed* (the *gas-liquid* mixture)

To *step off stages*, we *start from* the **intersection** of the **operating line** and the $\mathbf{X} = \boldsymbol{X}_{\boldsymbol{o}}$ **line**

Note that, for *stripping*, the *equilibrium* curve is *above* the *operating* line; this is because the **material** (*i.e.* a **gas** component **in** the **gas-liquid mixture**) *migrates from* the *liquid* phase to the *gas* phase [which is the same as the more volatile component ($MVC$) *moves from* the *liquid* phase to the *gas* phase in the *distillation*)

Accordingly, for the *stripping* operation, the **maximum amount** of **stripping gas** or the maximum $\dfrac{L}{G}$ can be determined

This can be done by drawing a *straight* line from the point of $\left(X_{N}, Y_{N+1}\right)$ until it *touches* the *equilibrium curve* at the point where X $= X_{o}$

However, it is *important* to note that this *straight line* (or the *maximum* $\frac{L}{G}$ *line*) cannot be *over* (or *higher than*) the *equilibrium line*; thus, *sometimes*, the maximum $\frac{L}{G}$ line is the *tangential* (but **still** *straight*) line to the *equilibrium* line that originates from the point $\left(X_{N}, Y_{N+1}\right)$ to the point where $X = X_{o}$, as illustrated as the *dashed lines* in Figure 8.6

**Example** We wish to design a *stripping* column to remove carbon dioxide ($CO_2$) from water

This can be done by heating the water + $CO_2$ mixture and passing it counter-currently with a nitrogen stream in a stripper

The operation is isothermal and isobaric at 60 °C and 1 atm

The *carbonated* water contains $9.2 \times 10^{-6}$ mole fraction of $CO_2$ and flows at 100,000 $\text{lb}_\text{m}$/h

The *nitrogen* stream enters the column as *pure* $N_2$ at 1 atm and 60 °C with the volumetric flow rate of 2,500 $\text{ft}^3$/h

Assume that $N_2$ is not dissolved in water and that water is not evaporated

Given the Henry's constant for $CO_2$ in water at 60 °C of 3,410 atm/(mole fraction)

If we desire an outlet water concentration of $2.0 \times 10^{-7}$ mole fraction of $CO_2$, **find the *number* of equilibrium *stages* required**

The volume of 1 lb-mol of nitrogen ($N_2$) at 1 atm (14.7 psia) and 60 °C (or 140 °F or 140 + 460 = 600 R) can be calculated, using the equation of state (EoS) for an ideal gas, as follows

$$PV = nRT$$

$$V = \frac{nRT}{P}$$

$$V = \frac{\left(1 \text{ lb-mol}\right)\left[10.73 \ \frac{\left(\text{ft}^3\right)\left(\text{psia}\right)}{\left(\text{lb-mol}\right)\left(\text{R}\right)}\right]\left[\left(140 + 460\right) \ \text{R}\right]}{\left(14.7 \text{ psia}\right)}$$

$$V = 438 \text{ ft}^3$$

In other words, we can say that the specific volume $\left(v\right)$ of $N_2$ is

$$v = \frac{V}{m} = \frac{438 \text{ ft}^3}{1 \text{ lb-mol}} = 438 \ \frac{\text{ft}^3}{\text{lb-mol}}$$

It is given that the *volumetric* flow rate of $N_2$ is 2,500 $\text{ft}^3$/h, which can be converted to *molar* flow rate as follows

$$\frac{2,500 \ \dfrac{\text{ft}^3}{\text{h}}}{438 \ \dfrac{\text{ft}^3}{\text{lb-mol}}} = 5.71 \ \frac{\text{lb-mol}}{\text{h}}$$

The flow rate of the *carbonated* water is given as 100,000 $\text{lb}_\text{m}$/h

With the molecular weight of water of 18.02 $\text{lb}_\text{m}$/lb-mol, the molar flow rate of the *carbonated* water can be computed as follows

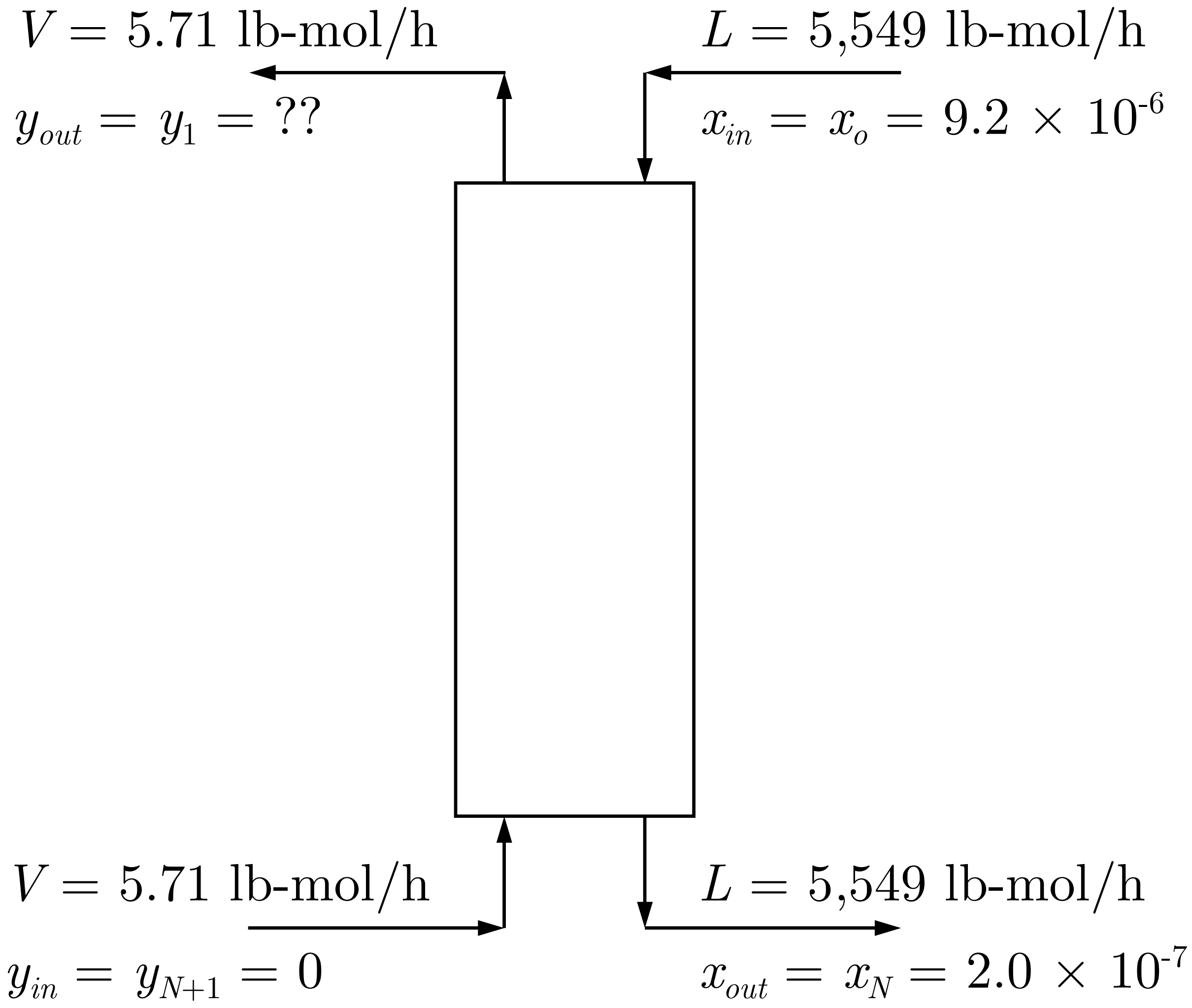
$$\frac{100{,}000\ \dfrac{\text{lb}_\text{m}}{\text{h}}}{18.02\ \dfrac{\text{lb}_\text{m}}{\text{lb-mol}}} = 5{,}549\ \frac{\text{lb-mol}}{\text{h}}$$

It is important to note that, since the amount of $CO_2$ in the *carbonated* water is *extremely low*, it is reasonable to assume that the flow rate of the *carbonated* water (*i.e.* the mixture of water + $CO_2$) is about the same as the flow rate of *pure* water (which is found to be 5,549 lb-mol/h)

It is given, in the problem statement, that

- $x_{in} = x_o = 9.2 \times 10^{-6} = 92 \times 10^{-7}$
- $x_{out} = x_N = 2.0 \times 10^{-7}$
- $y_{in} = y_{N+1} = 0 \; (pure \; \mathrm{N_2})$

The schematic diagram for *stripping* can be presented as shown below



Performing a species balance for $CO_2$:

$$Vy_{N+1} + Lx_o = Vy_1 + Lx_N \qquad (8.12)$$

gives

$$\left(5.71\right)\left(0\right) + \left(5,549\right)\left(9.2 \times 10^{-6}\right) = \left(5.71\right)y_1 + \left(5,549\right)\left(2.0 \times 10^{-7}\right)$$

$$y_1 \approx 0.00875$$

Note that, as the *concentration* of the *solute* is *extremely low*, the *flow rates* of the *liquid* and the *gas* phases can be assumed to be *constant* and the $x$ and $y$ co-ordinate can be used

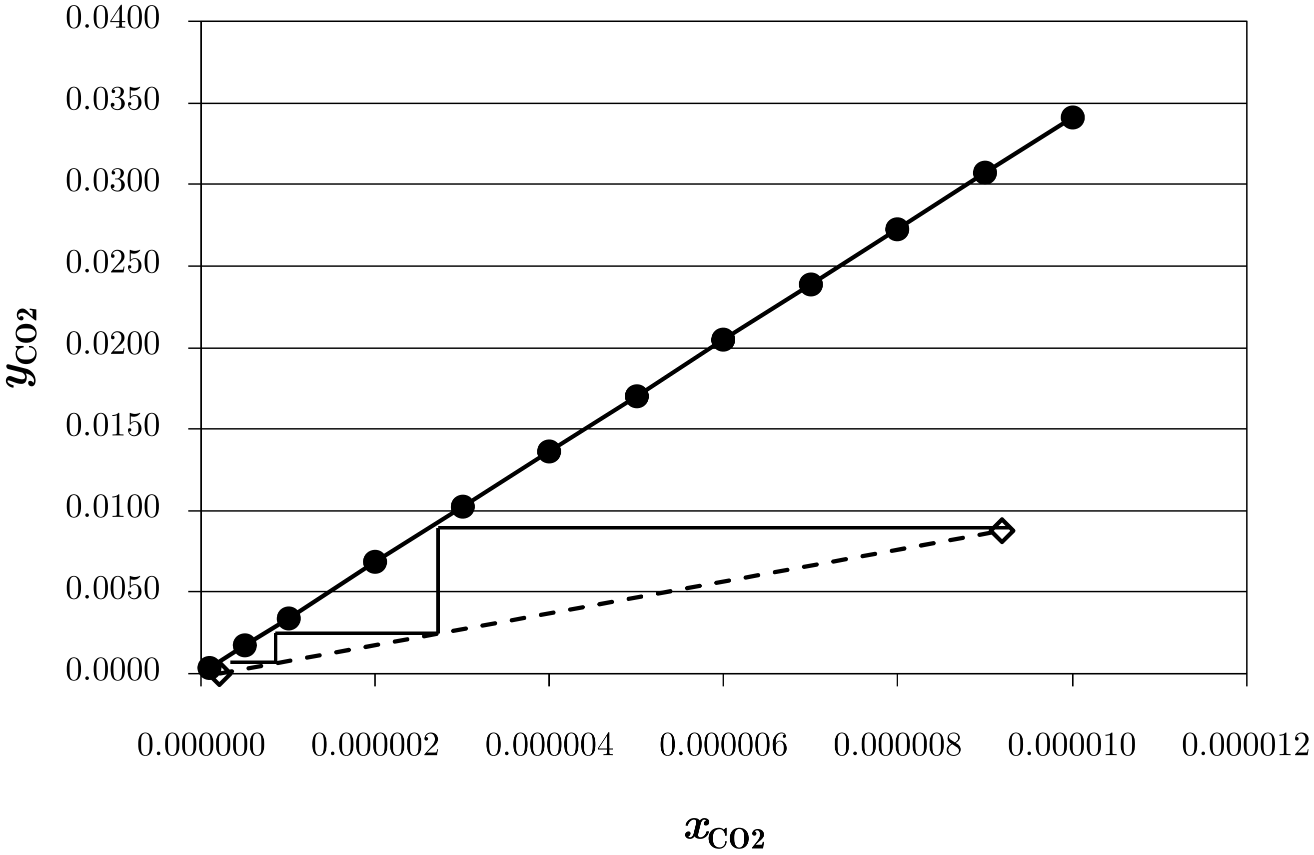
Accordingly, the **operating line** (dashed lines) passes through two points: $\left(x_N, y_{N+1}\right)$ and $\left(x_o, y_1\right)$ or $\left(2.0\times 10^{-7}, 0\right)$ and $\left(9.2\times 10^{-6}, 0.00875\right)$

The **equilibrium line** (solid line) can be drawn from the equilibrium equation as follows:

$$y = \frac{3,410 \dfrac{\text{atm}}{\left(\text{mole fraction}\right)}}{1 \text{ atm}} x$$

$$y = 3,410x$$

By drawing the *operating line* and the *equilibrium line* on the same McCabe-Thiele diagram, we can step off stages, which, in this Question, is found to be **~3** (note that, since this is the *stripping* operation, the *equilibrium* line is *above* the *operating* line)



## 8.4 Analytical Solution: Kremser Equation

When the *concentration* of a *solute* in both *gas* and *liquid* phases is very *low* ($< 1\%$), the total **gas** and **liquid flow rates** do **not change significantly**, even though the solute is transferred from the gas phase to the liquid phase

Thus, the **mole fractions** of **species *i*** in both gas $\left(y_i\right)$ and liquid $\left(x_i\right)$ phases **can be used** for the **calculations** (in other words, it is **NOT** *necessary* to *convert* that $y$ and $x$ data to the $Y$ and $X$ co-ordinate)

For the *dilute* situation described above, Figure 8.2 (on Page 13), which is on $X$ and $Y$ basis can be replaced by Figure 8.6

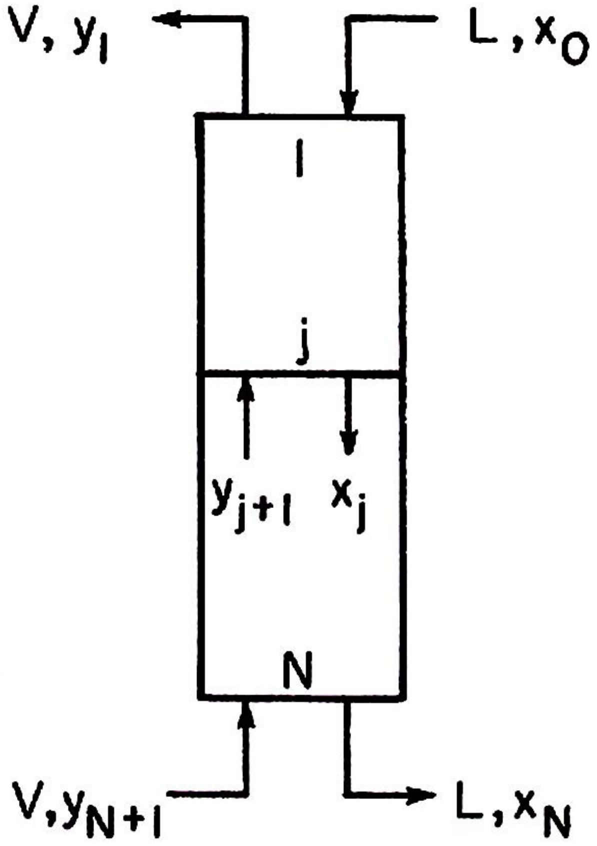


**Figure 8.6: A gas absorption when the concentration of a solute in both gas and liquid phases are low**

(from "Separation Process Engineering" by Wankat, 2007)

For a *dilute* absorber, the operating line is *similar* to that of the *normal* absorption operation (or Eq. 8.10), except that

- $Y$ is replaced by $y$ (*i.e.* the mole fraction of a solute in the *gas* phase)
- $X$ is replaced by $x$ (*i.e.* the mole fraction of a solute in the *liquid* phase)
- $G$ is replaced by $V$

Thus, the operating line for the dilute absorption can be written as follows

$$y_{j+1} = \frac{L}{V} x_j + \left( y_1 - \frac{L}{V} x_o \right) \tag{8.13}$$

All of the **assumptions** are **still** the **same** as *per* the *normal* absorption (see Page 7), with an **additional assumption** that the **concentration** of the **solute** in both *gas* and *liquid* phases is **very low**

To enable the *stage-by-stage* problem to be solved *analytically*, an ***additional*** **assumption** must be made, and it is the assumption that the **equilibrium line** is ***linear***; *i.e.*

$$y_j = mx_j + b \tag{8.14}$$

Actually, the **Henry's law** equation (Eq. 8.4):

$$y_j = \frac{H_B}{P_{\text{total}}} x_j$$

already **satisfies** this **assumption**, especially with the assumption that this **system** is ***isobaric***

By comparing the Henry's law equation with Eq. 8.14, it results in the fact that

- $m = \dfrac{H_B}{P_{\text{total}}}$
- $b = 0$

An **analytical** solution for the **absorption** operation can be derived for 2 special cases:

- When the **operating** and **equilibrium** lines are **parallel** to each other (*i.e.* $\dfrac{L}{V} = m$), as illustrated in Figure 8.7
- When the **operating** and **equilibrium** lines are **NOT parallel** to each other (*i.e.* $\dfrac{L}{V} < m$), as shown in Figure 8.8

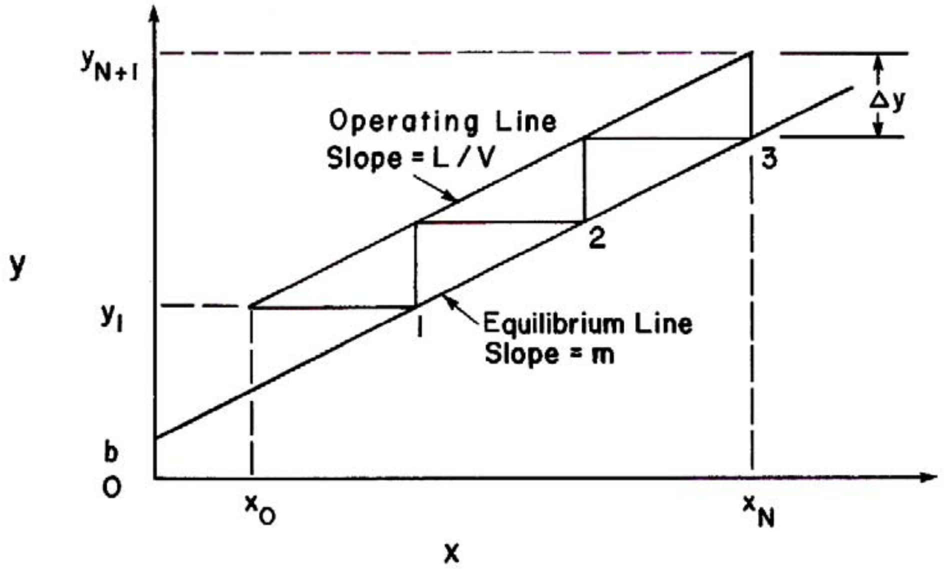


**Figure 8.7: The absorption operation for the case that the operating and equilibrium lines are parallel to each other**

(from "Separation Process Engineering" by Wankat, 2007)

For the case that the *operating* and *equilibrium* lines are *parallel* to each other or when $\frac{L}{V} = m$, we obtain the fact, for the absorber with $N$ equilibrium stages, that

$$y_{N+1} - y_1 = N\Delta y \tag{8.15}$$

where

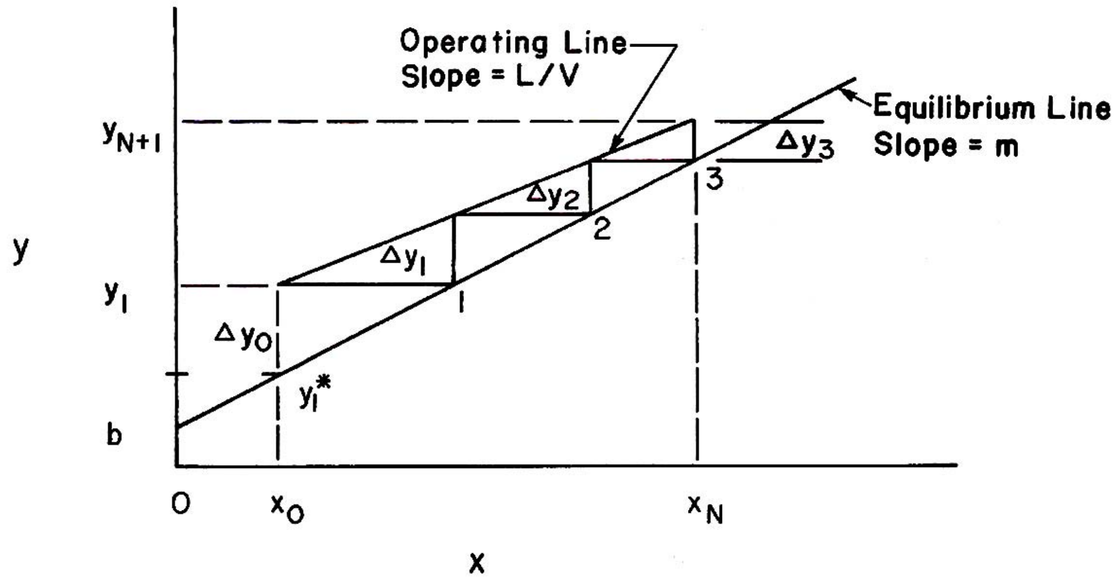


**Figure 8.8: The absorption operation for the case that the operating and equilibrium lines are not parallel to each other**

(from "Separation Process Engineering" by Wankat, 2007)

$$\Delta y = y_{j+1} - y_j \tag{8.16}$$

in which

- $y_{j+1}$ is obtained from the *operating* line (Eq. 8.13)
- $y_j$ is obtained from the *equilibrium* line (Eq. 8.14)

Combining Eqs. 8.13 & 8.14 with Eq. 8.16 and re-arranging yields

$$\Delta y = \left(\frac{L}{V} - m\right)x_j + \left(y_1 - \frac{L}{V}x_o - b\right) \tag{8.17}$$

Since, in this case, $\dfrac{L}{V} = m$, Eq. 8.17 becomes

$$\Delta y = \left(y_1 - \frac{L}{V}x_o - b\right) = \text{constant} \tag{8.18}$$

Note that Eq. 8.18 is true because the *operating* and *equilibrium* lines are *parallel* to each other; thus, the *distance* between the *operating* and the *equilibrium* lines or $\boldsymbol{\Delta y}$ are ***constant***

Substituting Eq. 8.18:

$$\Delta y = \left( y_1 - \frac{L}{V} x_o - b \right) \tag{8.18}$$

into Eq. 8.15:

$$y_{N+1} - y_1 = N \Delta y \tag{8.15}$$

and re-arranging the resulting equation yields

$$N \left( y_1 - \frac{L}{V} x_o - b \right) = y_{N+1} - y_1$$

$$N = \frac{y_{N+1} - y_1}{\left( y_1 - \frac{L}{V} x_o - b \right)} \tag{8.19}$$

**Eq. 8.18** is a **special case** of the **Kremser equation**, when $\frac{L}{V} = m$ $\left( \text{or } \frac{L}{mV} = 1 \right)$

For the case where $\frac{L}{V} < m$ (see Figure 8.8 on Page 51), $\Delta y$ is *no longer constant* or $\Delta y$ *varies from stage to stage*

Re-arranging Eq. 8.14:

$$y_j = mx_j + b \tag{8.14}$$

results in

$$x_j = \frac{y_j - b}{m} \tag{8.20}$$

Substituting Eq. 8.20 into Eq. 8.17:

$$\Delta y = \left(\frac{L}{V} - m\right)x_j + \left(y_1 - \frac{L}{V}x_o - b\right) \tag{8.17}$$

and re-arranging for $\Delta y$ of the stages $j$ and $j+1$ yields

$$\left(\Delta y\right)_j = \left(\frac{L}{mV} - 1\right) y_j + \left(y_1 - \frac{L}{mV} b - \frac{L}{V} x_o\right) \tag{8.21}$$

and

$$\left(\Delta y\right)_{j+1} = \left(\frac{L}{mV} - 1\right) y_{j+1} + \left(y_1 - \frac{L}{mV} b - \frac{L}{V} x_o\right) \tag{8.22}$$

(8.22) – (8.21) and re-arranging gives

$$\left(\Delta y\right)_{j+1} = \frac{L}{mV} \left(\Delta y\right)_j \tag{8.23}$$

Eq. 8.23 provides the relationship of $\Delta y$ for the *adjacent* stages (stage ที่ติดกัน; *e.g.*, stage 1 and stage 2) with the coefficient of $\frac{L}{mV}$ (commonly called the **"absorption factor"**)

Since $\Delta y$ is **NOT constant**, Eq. 8.15 is written as

$$y_{N+1} - y_1 = \Delta y_1 + \Delta y_2 + \Delta y_3 ... + \Delta y_N \tag{8.24}$$

By employing Eq. 8.23, we obtain the fact that, *e.g.*,

$$\left(\Delta y\right)_2 = \frac{L}{mV}\left(\Delta y\right)_1 \tag{8.25}$$

$$\left(\Delta y\right)_3 = \frac{L}{mV}\left(\Delta y\right)_2 \tag{8.26}$$

Substituting Eq. 8.25 into Eq. 8.26 yields

$$\left(\Delta y\right)_3 = \frac{L}{mV}\left[\frac{L}{mV}\left(\Delta y\right)_1\right]$$

$$\left(\Delta y\right)_3 = \left(\frac{L}{mV}\right)^2 \left(\Delta y\right)_1 \tag{8.27}$$

Eq. 8.27 can be written in a general form as

$$\left(\Delta y\right)_{j+1} = \left(\frac{L}{mV}\right)^{j}\left(\Delta y\right)_{1} \tag{8.28}$$

By applying Eq. 8.28 with Eq. 8.24:

$$y_{N+1} - y_1 = \Delta y_1 + \Delta y_2 + \Delta y_3 ... + \Delta y_N \tag{8.24}$$

we obtain the following equation:

$$y_{N+1} - y_1 = \left[1 + \left(\frac{L}{mV}\right) + \left(\frac{L}{mV}\right)^{2} + ... + \left(\frac{L}{mV}\right)^{N-1}\right]\Delta y_1 \tag{8.29}$$

After performing mathematical manipulation for the summation on the right hand side (RHS) of Eq. 8.29 and re-arranging the resulting equation, it results in

$$y_{N+1} - y_1 = \left[ \frac{1 - \left( \frac{L}{mV} \right)^N}{1 - \left( \frac{L}{mV} \right)} \right] \Delta y_1$$

or

$$\frac{y_{N+1} - y_1}{\Delta y_1} = \left[ \frac{1 - \left( \frac{L}{mV} \right)^N}{1 - \left( \frac{L}{mV} \right)} \right] \tag{8.30}$$

Writing Eq. 8.23:

$$\left( \Delta y \right)_{j+1} = \frac{L}{mV} \left( \Delta y \right)_j \tag{8.23}$$

for stage 1 gives

$$\left( \Delta y \right)_{o+1} = \frac{L}{mV} \left( \Delta y \right)_o$$

or

$$\left(\Delta y\right)_1 = \frac{L}{mV}\left(\Delta y\right)_o \tag{8.31}$$

where $\left(\Delta y\right)_o = \Delta y \ \left(\text{at } x = x_o\right) = y_1 - y_1^*$ (see Figure 8.8 on Page 51)

Note that, from the equilibrium line/equation,

$$y_1^* = mx_o + b \tag{8.32}$$

Thus, Eq. 8.31 can be re-written as

$$\left(\Delta y\right)_1 = \frac{L}{mV}\left(y_1 - y_1^*\right) \tag{8.33}$$

Substituting Eq. 8.33 into Eq. 8.30 and rearranging gives

$$\frac{y_{N+1} - y_1}{\frac{L}{mV}\left(y_1 - y_1^*\right)} = \frac{1 - \left(\frac{L}{mV}\right)^N}{1 - \left(\frac{L}{mV}\right)}$$

$$\frac{y_{N+1}-y_1}{\left(y_1-y_1^*\right)}=\frac{\dfrac{L}{mV}-\left(\dfrac{L}{mV}\right)^{N+1}}{1-\left(\dfrac{L}{mV}\right)} \tag{8.34}$$

Solving Eq. 8.34 for $N$ yields

$$N=\frac{\ln\left[\left(1-\dfrac{mV}{L}\right)\left(\dfrac{y_{N+1}-y_1^*}{y_1-y_1^*}\right)+\dfrac{mV}{L}\right]}{\ln\left(\dfrac{mV}{L}\right)} \tag{8.35}$$

(note that, in this case, $\dfrac{L}{V}\neq m$ or $\dfrac{L}{mV}\neq 1$)

**Eq. 8.35** is **another** form of the **Kremser equation**, when $\dfrac{L}{mV}\neq 1$

The Kremser equations in terms of *gas-phase* composition can be written as follows

- $$\frac{y_{N+1} - y_1}{y_1 - y_1^*} = \frac{\left(\frac{L}{mV}\right) - \left(\frac{L}{mV}\right)^{N+1}}{1 - \left(\frac{L}{mV}\right)} \qquad (8.34)$$
- $$\frac{y_{N+1} - y_{N+1}^*}{y_1 - y_1^*} = \left(\frac{L}{mV}\right)^N \qquad (8.36)$$
- $$N = \frac{\ln\left[\frac{\left(y_{N+1} - y_{N+1}^*\right)}{\left(y_1 - y_1^*\right)}\right]}{\ln\left(\frac{L}{mV}\right)} \qquad (8.37)$$
- $$N = \frac{\ln\left[\frac{\left(y_{N+1} - y_{N+1}^*\right)}{\left(y_1 - y_1^*\right)}\right]}{\ln\left[\frac{\left(y_{N+1} - y_1\right)}{\left(y_{N+1}^* - y_1^*\right)}\right]} \qquad (8.38)$$

where $y_{N+1}^* = mx_N + b$ and $y_1^* = mx_o + b$

The Kremser equations in terms of *liquid-phase* composition are

- $$N = \frac{\ln\left[\left(1 - \frac{L}{mV}\right)\left(\frac{x_o - x_N^*}{x_N - x_N^*}\right) + \frac{L}{mV}\right]}{\ln\left(\frac{mV}{L}\right)} \tag{8.39}$$

- $$N = \frac{\ln\left[\left(\frac{x_N - x_N^*}{x_o - x_o^*}\right)\right]}{\ln\left[\left(\frac{x_o^* - x_N^*}{x_o - x_N}\right)\right]} \tag{8.40}$$

- $$\frac{x_N - x_N^*}{x_o - x_N^*} = \frac{1 - \left(\frac{mV}{L}\right)}{1 - \left(\frac{mV}{L}\right)^{N+1}} \tag{8.41}$$

- $$\frac{x_N - x_N^*}{x_o - x_o^*} = \left(\frac{L}{mV}\right)^N \qquad (8.42)$$

where $x_N^* = \dfrac{y_{N+1} - b}{m}$ and $x_o^* = \dfrac{y_1 - b}{m}$

Among many equations above, how to choose the appropriate equation(s) depends on the data given in the problem statement

Let's examine the following Example, which illustrates how to select the proper Kremser equation for a specified problem

**Example** A plate tower with 6 equilibrium stages is employed for *stripping* ammonia from waste water with the **inlet** concentration of **0.10 mol%** using counter-current air at atmospheric pressure and 80 °F

Determine the concentration of ammonia in the **exit** waste water if the *stripping* air is *ammonia-free* and the feed rate of air is 30 standard cubic feet (scf) *per* 1 $\text{lb}_\text{m}$ of waste water

Given the equilibrium equation of ammonia at 80 °F as $y = 1.414x$

One (1) lb-mol of air is equivalent to the **volume** $\left(\bar{V}\right)$ of 379 standard cubic feet (scf) [for 60 °F (519.67 R) and 14.7 psi ($\text{lb}_\text{f}/\text{in}^2$)], which can be computed as follows

$$\bar{V} = \frac{nRT}{P}$$

$$= \frac{\left(1 \text{ lb-mol}\right)\left[10.73 \ \frac{\left(\text{psia}\right)\left(\text{ft}^3\right)}{\left(\text{lb-mol}\right)\left(\text{R}\right)}\right]\left[\left(60 + 459.67\right) \text{ R}\right]}{\left(14.7 \text{ psia}\right)}$$

$$\bar{V} = 379 \text{ ft}^3 \ \left(\text{scf}\right)$$

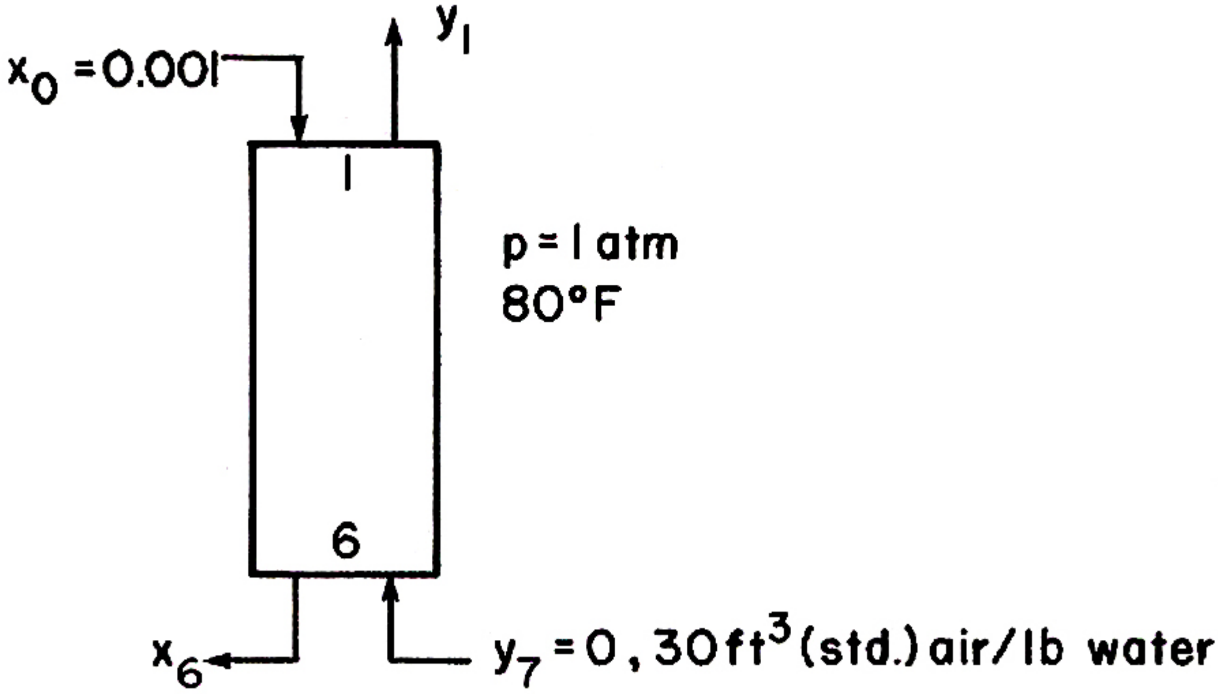
Thus, the molar flow rate of the stripping gas $\left(V\right)$ is

$$V = \frac{30 \text{ scf air}}{379 \ \frac{\text{scf air}}{\text{lb-mol air}}} = 0.0792 \text{ lb-mol air}$$

The molecular weight (MW) of water is 18.02 $\frac{\text{lb}_\text{m}}{\text{lb-mol}}$; hence, the molar flow rate of the waste water $\left(L\right)$ can be calculated as follows

$$L = \frac{1\ \text{lb}_\text{m}\ \text{water}}{18.02\ \dfrac{\text{lb}_\text{m}\ \text{water}}{\text{lb-mol water}}} = 0.0555\ \text{lb-mol water}$$

The given data are (see the Figure below)



(from “Separation Process Engineering” by Wankat, 2007)

- The inlet concentration of ammonia ($NH_3$) in the waste water: $x_o = \dfrac{0.1}{100} = 0.001$

- The inlet concentration of $NH_3$ in the stripping gas or air: $y_{N+1} = y_7 = 0$ (as the air is *ammonia-free*)
- $m = 1.414$ and $b = 0$ [from the given equilibrium equation: $y = 1.414x$, compared to the standard equilibrium equation (Eq. 8.14): $y = mx + b$]
- The number of equilibrium stages $\left(N\right) = 6$

Thus, by employing Eq. 8.41:

$$\frac{x_N - x_N^*}{x_o - x_N^*} = \frac{1 - \left(\frac{mV}{L}\right)}{1 - \left(\frac{mV}{L}\right)^{N+1}} \tag{8.41}$$

with the fact that

$$x_N^* = \frac{y_{N+1} - b}{m}$$

which yields

$$x_N^* = \frac{0-0}{1.414} = 0$$

we can compute the value of the **outlet concentration** of **NH$_3$** in the **waste water** ($x_N$ or $x_6$) as follows

$$x_N - x_N^* = \left[\frac{1-\dfrac{mV}{L}}{1-\left(\dfrac{mV}{L}\right)^{N+1}}\right]\left(x_o - x_N^*\right)$$

$$x_N - 0 = \left[\frac{1-\dfrac{\left(1.414\right)\left(0.0792\right)}{\left(0.0555\right)}}{1-\left[\dfrac{\left(1.414\right)\left(0.0792\right)}{\left(0.0555\right)}\right]^{6+1}}\right]\left(0.0010-0\right)$$

$$x_N = 7.45\times 10^{-6} = 7.45 \text{ ppm}$$