AE 335 Separation Processes

(by PTS)

Solutions to Problem Set 4

(Multi-component Distillation)

1. For five-component distillation with one LK, one HK, one HNK, and 2 LNKs, where is the *lowest* temperature in the column?

   At the **top** of the **column**

2. A distillation column is separating methane, ethane, propane, and butane.

   2.1) If methane and propane are selected as the *key* components, what are ethane and butane?

   Ethane = a **sandwich** component
   Butane = a **heavy non-key (HNK)** component

   2.2) If propane and butane are selected as the *key* components, what are methane and ethane?

   Both methane and ethane = **heavy non-key (HNK)** components

3. We are separating the mixture of 10 mol% methanol, 20% ethanol, 30% *n*-propanol, and 40% *n*-butanol using a distillation column. We desire to recover 98% of ethanol in the distillate and 97% *n*-propanol in the bottom product

   3.1) What are the heavy key and the light key components, respectively?

   Ethanol and ***n*-propanol** = **light key (LK)** and **heavy key (HK)** components, respectively

   3.2) Specify the light non-key(s), if any (ถ้ามี)

   Methanol = a **light non-key (LNK)** species

   3.3) Specify the heavy non-key(s), if any (ถ้ามี)

   n-butanol = a **heavy non-key (HNK)** substance

3.4) What is the *best* assumption we should make if we are going to perform the external mass balance? (by selecting an appropriate answer from the following choices)

a) All of methanol and *n*-propanol are in the distillate
b) All of methanol is in the distillate and all of *n*-butanol is in the bottom product
c) All of ethanol is in the distillate and all of *n*-propanol is in the bottom product
d) All of the *n*-propanol and *n*-butanol are in the bottom product

The answer is ***b***, as it is common to make assumptions that all the LNKs appear only in the distillate, while all the HNKs are in the bottom product only

4. A mixture of 60 wt% benzene, 35% toluene, and 5% naphthalene is being distilled. The distillate product should contain 99.5 wt% benzene. Also, 99% of benzene fed should be recovered in the distillate. Determine/calculate:

4.1) distillate and bottom product flow rates (D and B) *per* 1 kg of feed
4.2) compositions of distillate and bottom product
4.3) fraction of toluene fed that is recovered in the bottom product

**** Note that, in this Question, all the calculations are on MASS basis, which is O.K. as there are no reactions within the distillation column ****

It is given that the fractional recovery of benzene in the distillate $\left[(\mathrm{FR})_{\text{benzene, dist.}}\right]$ is 99%

This means that

$$\frac{\text{Mass of benzene in the distilate}}{\text{Mass of bezene in the feed}} \times 100 = 99$$

From the given data, 1 kg of feed contains benzene in the amount of 0.60 kg (*i.e.* there are 60 wt% of benzene in the feed)

Thus, mass of benzene in the distillate $\left(Dx_{D,\text{ benzene}}\right)$ is

$$Dx_{D,\text{ benzene}} = \left(\frac{99}{100}\right)(0.60) = 0.594 \text{ kg}$$

It is also given that the distillate has a concentration of benzene $\left(x_{D,\text{ benzene}}\right)$ of 99.5%, or

$$x_{D,\text{ benzene}} = 0.995$$

Thus, the amount (mass) of the distillate (D) is

$$D = \frac{Dx_{D,\text{ benzene}}}{x_{D,\text{ benzene}}} = \frac{0.594}{0.995} = 0.597\text{ kg}$$

Since $D + B = \text{Feed} = 1\text{kg}$, the amount (mass) of the bottom product (B) is

$$B = \text{Feed} - D = 1 - 0.597 = 0.403\text{ kg}$$

As the fractional recovery of benzene is specified in the problem statement, benzene is the key component, and, since benzene is the most volatile component, it is the LK

Assume that toluene is the HK; thus, naphthalene is the HNK

As naphthalene is the HNK, it can be assumed that it appears only in the bottom (*i.e.* it does not appear in the distillate), or

$$x_{D,\text{ naphthalene}} = 0$$

Since

$$\sum x_{D,\,i} = x_{D,\text{ benzene}} + x_{D,\text{ toluene}} + x_{D,\text{ naphthalene}} = 1.0$$

the concentration of toluene in the distillate $\left(x_{D,\text{ toluene}}\right)$ is

$$\begin{aligned} x_{D,\text{ toluene}} &= 1.0 - x_{D,\text{ benzene}} - x_{D,\text{ naphthalene}} \\ &= 1.0 - 0.995 - 0 \\ x_{D,\text{ toluene}} &= 0.005 \end{aligned}$$

For the bottom product, since the fractional recovery of benzene in the distillate $\left[(\text{FR})_{\text{benzene, dist.}}\right]$ is given as 99%, the fractional recovery of benzene in the bottom product is

100 – 99 = 1% or 0.01

Hence,

$$Bx_{B,\text{ benzene}} = (\text{FR})_{\text{benzene, bottom}} Fz_{F,\text{ benzene}}$$

and

$$\begin{aligned} x_{B,\text{ benzene}} &= \frac{(\text{FR})_{\text{benzene, bottom}} Fz_{F,\text{ benzene}}}{B} \\ &= \frac{(0.01)(1)\left(\frac{60}{100}\right)}{0.403} \\ x_{B,\text{ benzene}} &= 0.0149 \end{aligned}$$

We have just assume that all of naphthalene appears in the bottom (because it is assumed to be the HNK), which means that

$$Bx_{B,\text{ naphthalene}} = Fz_{F,\text{ naphthalene}}$$

Thus,

$$\begin{aligned} x_{B,\text{ naphthalene}} &= \frac{Fz_{F,\text{ naphthalene}}}{B} \\ &= \frac{(1)\left(\frac{5}{100}\right)}{0.403} \\ x_{B,\text{ naphthalene}} &= 0.1241 \end{aligned}$$

Accordingly,

$$\begin{aligned} x_{B,\text{ toluene}} &= 1.0 - x_{B,\text{ benzene}} - x_{B,\text{ naphthalene}} \\ &= 1.0 - 0.0149 - 0.1241 \\ x_{B,\text{ toluene}} &= 0.8610 \end{aligned}$$

The fraction recovery of toluene in the bottom $\left[(\text{FR})_{\text{toluene, bottom}}\right]$ means

$$\frac{\text{Mass of toluene in the bottom}}{\text{Mass of toluene in the feed}}$$

or

$$(\text{FR})_{\text{toluene, bottom}} = \frac{Bx_{B,\text{ toluene}}}{Fz_{F,\text{ toluene}}}$$

Hence,

$$(\text{FR})_{\text{toluene, bottom}} = \frac{(0.403)(0.8610)}{(1)\left(\frac{35}{100}\right)} = 0.9914 = 99.14\%$$

5. The 10,000 kmol/day of the mixture of 22 mol% methanol, 47% ethanol, 18% propanol, and 13% *n*-butanol is fed into a distillation column as a saturated liquid. We desire to have 99.8 mol% methanol recovered in the distillate and the mole fraction of methanol in the distillate is 0.99. Find
   5.1) D and B
   5.2) compositions of distillate and bottom products

**** Note that all the calculations in this Question are on MOLE basis ****

Since the fractional recovery of methanol in the distillate is specified (as 99.8 mol%), it is the key component, and, since it is the most volatile component, it is the LK

From the given data/information, we do not have any fractional recovery data of any other species, except that of methanol

Thus, for our convenience, we should assume that ethanol is the HK, so that all of HNKs, which are propanol and *n*-butanol, appear only in the bottom

From the given data of methanol (M), we obtain the fact that

$$\begin{aligned} Dx_{D,\,\mathrm{M}} &= (\mathrm{FR})_{\mathrm{M,\,dist}} Fz_{F,\,\mathrm{M}} \\ &= (0.998)(10{,}000)\left(\frac{22}{100}\right) \\ Dx_{D,\,\mathrm{M}} &= 2{,}195.6 \text{ kmol/h} \end{aligned}$$

Since it is given that $x_{D,\,\mathrm{M}} = 0.99$, the amount (molar flow rate) of the distillate (D) can be computed as follows

$$D = \frac{Dx_{D,\,\mathrm{M}}}{x_{D,\,\mathrm{M}}} = \frac{2{,}195.6}{0.99} = 2{,}218 \text{ kmol/h}$$

Thus, the amount (molar flow rate) of the bottom (B) is

$$B = 10{,}000 - D = 10{,}000 - 2{,}218 = 7{,}782 \text{ kmol/h}$$

From our assumption that propanol and *n*-butanol are the HNKs, there are no propanol (P) and *n*-butanol (B) in the distillate

Thus, the distillate contains only methanol (M) and ethanol (E)

Since $x_{D,\,\mathrm{M}} = 0.99$, the concentration of ethanol (E) in the distillate is

$$x_{D,\,\mathrm{E}} = 1.0 - 0.99 = 0.01$$

(note that, as we have assumed that proponol and n-butanol are the HNKs, they do not appear in the distillate; accordingly, the distillate contains only methanol and ethanol)

For the bottom product, since the fractional recovery of methanol in the distillate $\left[(\mathrm{FR})_{\mathrm{M,\,dist}}\right]$ is given as 99.8% or 0.998, its fractional recovery in the bottom $\left[(\mathrm{FR})_{\mathrm{M,\,bot}}\right]$ is

$$(\mathrm{FR})_{\mathrm{M,\,bot}} = 1 - 0.998 = 0.002$$

Thus, the amount (molar flow rate) of methanol in the bottom is

$$\begin{aligned} Bx_{B,\,\mathrm{M}} &= (\mathrm{FR})_{\mathrm{M,\,bot}} Fz_{F,\,\mathrm{M}} \\ &= (0.002)(10{,}000)\left(\frac{22}{100}\right) \\ Bx_{B,\,\mathrm{M}} &= 4.4 \text{ kmol/h} \end{aligned}$$

and the concentration of methanol in the bottom is

$$\begin{aligned} x_{B,\,\mathrm{M}} &= \frac{Bx_{B,\,\mathrm{M}}}{B} \\ &= \frac{4.4}{7{,}782} \\ x_{B,\,\mathrm{M}} &= 0.00057 \end{aligned}$$

Since propanol (P) and *n*-butanol (B) are assumed to be the HNKs, all amounts of these two substances in the feed appear only in the bottom, which means that

$$Bx_{B,\,\mathrm{P}} = Fz_{F,\,\mathrm{P}} = (10{,}000)\left(\frac{18}{100}\right) = 1{,}800 \text{ kmol/h}$$

and

$$Bx_{B,\,\mathrm{B}} = Fz_{F,\,\mathrm{B}} = (10{,}000)\left(\frac{13}{100}\right) = 1{,}300 \text{ kmol/h}$$

The concentrations of propanol (P) and *n*-butanol (B) can be computed as follows

$$\begin{aligned} x_{B,\text{ P}} &= \frac{Bx_{B,\text{ P}}}{B} \\ &= \frac{1{,}800}{7{,}782} \\ x_{B,\text{ P}} &= 0.231 \end{aligned}$$

and

$$\begin{aligned} x_{B,\text{ B}} &= \frac{Bx_{B,\text{ B}}}{B} \\ &= \frac{1{,}300}{7{,}782} \\ x_{B,\text{ B}} &= 0.167 \end{aligned}$$

Accordingly, the concentration of ethanol (E) in the bottom is

$$x_{B,\text{ E}} = 1.0 - x_{B,\text{ M}} - x_{B,\text{ P}} - x_{B,\text{ B}} = 1.0 - 0.00057 - 0.231 - 0.167 \approx 0.601$$

6. We are separating a mixture of 40 mol% *iso*-pentane, 30% *n*-hexane, and 30% *n*-heptane fed into the distillation column at the feed rate of 1,000 kmol/h. We desire a 98 mol% recovery of *n*-hexane in the bottom product and a 99% recovery of *iso*-pentane in the distillate. The feed is a two-phase mixture with 40% vapour, and the reflux ratio $\left(\frac{L}{D}\right)$ is 2.5
   6.1) Find D and B
   6.2) Find compositions of distillate and bottom products
   6.3) Calculate the values of L, V, $\bar{L}$, and $\bar{V}$, assuming that CMO is valid

From the problem statement,

- *iso*-pentane = LK
- *n*-hexane = HK

(try finding the answer yourself "why"?)

Thus, *n*-heptane = HNK

From the given data, we can calculate the following

$$Dx_{D,\text{ C5}} = (\text{FR})_{\text{C5, dist}} Fz_{F,\text{ C5}} = (0.99)(1{,}000)\left(\frac{40}{100}\right) = 396 \text{ kmol/h}$$

and

$$Bx_{B,\text{ C6}} = (\text{FR})_{\text{C6, bot}} Fz_{F,\text{ C6}} = (0.98)(1{,}000)\left(\frac{30}{100}\right) = 294 \text{ kmol/h}$$

Additionally, we can also compute the following

$$Bx_{B,\text{ C5}} = (\text{FR})_{\text{C5, bot}} Fz_{F,\text{ C5}} = (1-0.99)(1{,}000)\left(\frac{40}{100}\right) = 4 \text{ kmol/h}$$

and

$$Dx_{D,\text{ C6}} = (\text{FR})_{\text{C6, dist}} Fz_{F,\text{ C6}} = (1-0.98)(1{,}000)\left(\frac{60}{100}\right) = 6 \text{ kmol/h}$$

As *n*-heptane is HNK, it appears only in the bottom, which means that

$$Bx_{B,\text{ C7}} = Fz_{F,\text{ C7}} = (1{,}000)\left(\frac{30}{100}\right) = 300 \text{ kmol/h}$$

and

$$x_{D,\text{ C7}} = 0 \text{ and } Dx_{D,\text{ C7}} = 0$$

The amounts of the distillate (D) and the bottom (B) can be calculated as follows

$$D = \sum Dx_{D,\text{ i}} = Dx_{D,\text{ C5}} + Dx_{D,\text{ C6}} + Dx_{D,\text{ C7}}$$
$$= 396 + 6 + 0$$
$$D = 402 \text{ kmol/h}$$

and

$$B = \sum Bx_{B,\text{ i}} = Bx_{B,\text{ C5}} + Bx_{B,\text{ C6}} + Bx_{B,\text{ C7}}$$
$$= 4 + 294 + 300$$
$$B = 598 \text{ kmol/h}$$

The compositions of the distillate and the bottom products can be computed as follows

Distillate:

$$x_{D,\text{ C5}} = \frac{Dx_{D,\text{ C5}}}{D} = \frac{396}{402} = 0.985$$
$$x_{D,\text{ C6}} = \frac{Dx_{D,\text{ C6}}}{D} = \frac{6}{402} = 0.015$$
$$x_{D,\text{ C7}} = \frac{Dx_{D,\text{ C7}}}{D} = \frac{0}{402} = 0$$

Bottom:

$$x_{B,\text{ C5}} = \frac{Bx_{B,\text{ C5}}}{B} = \frac{4}{598} = 0.007$$
$$x_{B,\text{ C6}} = \frac{Bx_{B,\text{ C6}}}{B} = \frac{294}{598} = 0.491$$
$$x_{B,\text{ C7}} = \frac{Bx_{B,\text{ C7}}}{B} = \frac{300}{598} = 0.502$$

Since $L = \left(\frac{L}{D}\right)D$, the liquid flow rate for the rectifying/enriching section is

$$L = (2.5)(402) = 1{,}005 \text{ kmol/h}$$

Performing the balance around the condenser gives

$$V = L + D$$

Thus,

$$V = 1{,}005 + 402 = 1{,}407 \text{ kmol/h}$$

It is given that the 40% of the feed is in *vapour* phase, which means that

$$V_F = 0.40F = 0.40(1{,}000) = 400 \text{ kmol/h}$$

and that

$$L_F = 0.60F = 0.60(1{,}000) = 600 \text{ kmol/h}$$

We have learned that

$$V = \bar{V} + V_F$$

and that

$$\bar{L} = L + L_F$$

Thus,

$$\bar{V} = V - V_F = 1{,}407 - 400 = 1{,}007 \text{ kmol/h}$$

and

$$\bar{L} = 1{,}005 + 600 = 1{,}605 \text{ kmol/h}$$

7. A distillation column is separating methanol, ethanol, and butanol at a pressure of 2 atm. The column has a total condenser and a partial re-boiler. The feed is saturated liquid with the flow rate of 100 kmol/h comprising 45 mol% methanol, 30% ethanol, and 25% butanol. The reflux ratio $\left(\frac{L}{D}\right)$ is 2.0 and the reflux is a saturated liquid. We desire a 98% recovery of ethanol in the distillate and a 99.1% recovery of butanol in the bottom product. Find D and B, and compositions of distillate and bottom products

From the given information,

- ethanol (E) = LK
- butanol (B) = HK
- methanol (M) = LNK

(try figuring out yourself “why”?)

Thus, methanol appears only in the distillate

From the data of fractional recoveries of ethanol and butanol, which are

$$(\text{FR})_{\text{E, dist}} = 0.98 \ (98\%) \text{ or } (\text{FR})_{\text{E, bot}} = 1 - 0.98 = 0.02$$

and

$$(\text{FR})_{\text{B, bot}} = 0.991 \ (99.1\%) \text{ or } (\text{FR})_{\text{B, dist}} = 1 - 0.991 = 0.009$$

we can compute the following

<u>for ethanol (E)</u>:

$$Dx_{D,\text{ E}} = (\text{FR})_{\text{E, dist}} Fz_{F,\text{ E}} = (0.98)(100)\left(\frac{30}{100}\right) = 29.4 \text{ kmol/h}$$

$$Bx_{B,\text{ E}} = (\text{FR})_{\text{E, bot}} Fz_{F,\text{ E}} = (0.02)(100)\left(\frac{30}{100}\right) = 0.6 \text{ kmol/h}$$

and

<u>for butanol (B)</u>:

$$Bx_{B,\text{ B}} = (\text{FR})_{\text{B, bot}} Fz_{F,\text{ B}} = (0.991)(100)\left(\frac{25}{100}\right) = 24.78 \text{ kmol/h}$$

$$Dx_{D,\text{ B}} = (\text{FR})_{\text{B, dist}} Fz_{F,\text{ B}} = (0.009)(100)\left(\frac{25}{100}\right) = 0.225 \text{ kmol/h}$$

Additionally, as methanol (M) is the LNK, which appears only in the distillate (do you know "why"?); thus, we obtain the fact that

$$Dx_{D,\text{ M}} = Fz_{F,\text{ M}} = (100)\left(\frac{45}{100}\right) = 45\text{ kmol/h}$$

and

$$x_{B,\text{ M}} = 0 \text{ and } Bx_{B,\text{ M}} = 0$$

Hence,

$$D = \sum Dx_{D,\text{ i}} = 45 + 29.4 + 0.225 \approx 74.6\text{ kmol/h}$$
$$B = \sum Bx_{B,\text{ i}} = 0 + 0.6 + 24.78 \approx 25.4\text{ kmol/h}$$

The compositions of the distillate and the bottom products can, thus, be calculated as follows

Distillate:

$$x_{D,\text{ M}} = \frac{Dx_{D,\text{ M}}}{D} = \frac{45}{74.6} = 0.603$$
$$x_{D,\text{ E}} = \frac{Dx_{D,\text{ E}}}{D} = \frac{29.4}{74.6} = 0.394$$
$$x_{D,\text{ B}} = \frac{Dx_{D,\text{ B}}}{D} = \frac{0.225}{74.6} = 0.003$$

Bottom:

$$x_{B,\text{ M}} = \frac{Bx_{B,\text{ M}}}{B} = \frac{0}{25.4} = 0$$
$$x_{B,\text{ E}} = \frac{Bx_{B,\text{ E}}}{B} = \frac{0.6}{25.4} = 0.024$$
$$x_{B,\text{ B}} = \frac{Bx_{B,\text{ B}}}{B} = \frac{24.78}{25.4} = 0.976$$

8. We have 10 kmol/h of a saturated liquid that is composed of 40 mol% benzene (B) and 60% toluene (T). We desire a distillate composition with 99.2 mol% of benzene and a bottom product composition with 98.6 mol% of toluene. The relative volatility, α_{BT}, is 2.4. The reflux is returned as a saturated liquid, and the column has a total condenser and a partial re-boiler. Assume that CMO is valid.

   8.1) Use the Fenske equation to determine $N_{\min}$

$$N_{\min} = \frac{\ln\left\{\frac{\left[x/(1-x)\right]_{\text{dist}}}{\left[x/(1-x)\right]_{\text{bot}}}\right\}}{\ln\alpha_{AB}} = \frac{\ln\left\{\frac{\left[x/(1-x)\right]_{\text{dist}}}{\left[x/(1-x)\right]_{\text{bot}}}\right\}}{\ln\alpha_{BT}} = \frac{\ln\left\{\frac{\left[0.992/(1-0.992)\right]}{\left[0.986/(1-0.986)\right]}\right\}}{\ln(2.4)} = 10.36$$

Note that

- A ≡ benzene (B) – its concentration in the distillate is specified
- B ≡ toluene (T) – its concentration in the bottom product is specified

We can also compute the optimal feed stage for the minimum number of stage $\left(N_{F,\ \min}\right)$ as follows

$$N_{F,\ \min}=\frac{\ln\left\{\frac{\left[x_{\mathrm{LK}}/x_{\mathrm{HK}}\right]_{\mathrm{dist}}}{\left[z_{\mathrm{LK}}/z_{\mathrm{HK}}\right]_{\mathrm{feed}}}\right\}}{\ln\alpha_{\mathrm{LK-HK}}}=\frac{\ln\left\{\frac{\left[x_{\mathrm{B}}/x_{\mathrm{T}}\right]_{\mathrm{dist}}}{\left[z_{\mathrm{B}}/z_{\mathrm{T}}\right]_{\mathrm{feed}}}\right\}}{\ln\alpha_{\mathrm{BT}}}=\frac{\ln\left\{\frac{\left[0.992/\left(1-0.992\right)\right]_{\mathrm{dist}}}{\left[0.40/0.60\right]_{\mathrm{feed}}}\right\}}{\ln\left(2.4\right)}=5.97$$

(note that this counts from the top of the column)

Note that, since this system is a binary mixture, x_B+x_T in the feed, the distillate and the bottom = 1; thus, $x_{T,\ \mathrm{dist}}=1-x_{B,\ \mathrm{dist}}$ and $z_T=1-z_B$

8.2) Use the Underwood equations to find $\left(\frac{L}{D}\right)_{\min}$

The feed is saturated liquid; thus, ΔV_{feed} or $V_{F,\ \min}$ = 0

Thus,

$$\Delta V_{feed}=0=\sum\frac{\alpha_i F z_i}{\left(\alpha_i-\phi\right)}=\frac{\alpha_B F z_B}{\left(\alpha_B-\phi\right)}+\frac{\alpha_T F z_T}{\left(\alpha_T-\phi\right)}$$

Let toluene (T) a reference substance; thus

- $\alpha_B=\alpha_{BT}=\frac{K_B}{K_T}=2.4$
- $\alpha_T=\alpha_{TT}=\frac{K_T}{K_T}=1.0$

Hence, we obtain

$$0=\frac{\alpha_B F z_B}{\left(\alpha_B-\phi\right)}+\frac{\alpha_T F z_T}{\left(\alpha_T-\phi\right)}=\frac{\left(2.4\right)\left(10\right)\left(\frac{40}{100}\right)}{\left(2.4-\phi\right)}+\frac{\left(1.0\right)\left(10\right)\left(\frac{60}{100}\right)}{\left(1.0-\phi\right)}$$

Solving the above equation for ϕ yields

$$\phi=1.54$$

We have learned that

$$D=\left(\frac{z-x_{\mathrm{bot}}}{x_{\mathrm{dist}}-x_{\mathrm{bot}}}\right)F$$

By using the data of benzene to calculate the amount of the distillate $\left(D\right)$, we obtain

$$D=\left[\frac{0.40-\left(1-0.986\right)}{0.992-\left(1-0.986\right)}\right]\left(10\right)=3.95\ \mathrm{kmol/h}$$

Hence,

$$V_{\min} = \sum \frac{\alpha_i D x_{i,\text{ dist}}}{(\alpha_i - \phi)} = \frac{\alpha_B D x_{B,\text{ dist}}}{(\alpha_B - \phi)} + \frac{\alpha_T D x_{T,\text{ dist}}}{(\alpha_T - \phi)}$$

$$= \frac{(2.4)(3.95)(0.992)}{(2.4 - 1.54)} + \frac{(1.0)(3.95)(1 - 0.992)}{(1.0 - 1.54)}$$

$$V_{\min} = 10.85 \text{ kmol/h}$$

Since $V_{\min} = L_{\min} + D$, we obtain

$$L_{\min} = V_{\min} - D = 10.85 - 3.95 = 6.90 \text{ kmol/h}$$

Thus, the minimum reflux ratio $\left[\left(\frac{L}{D}\right)_{\min}\right]$ is

$$\frac{L_{\min}}{D} = \frac{6.90}{3.95} = 1.75$$

8.3) For actual $\frac{L}{D} = 1.1\left(\frac{L}{D}\right)_{\min}$, use the previous results (from 8.1 & 8.2) and the Gilliland correlation to estimate the total number of stages and optimal feed location

$$\frac{L}{D} = 1.1\left(\frac{L}{D}\right)_{\min} = 1.1 \times 1.75 = 1.925$$

Thus, the abscissa or the value of the X-axis is

$$x = \frac{(L/D) - (L/D)_{\min}}{(L/D) + 1} = \frac{1.925 - 1.75}{1.925 + 1} = 0.0598$$

As x = 0.0598, the ordinate or the value of the Y-axis can be computed from the following equation as follows

$$y = \frac{N - N_{\min}}{N + 1} = 0.545827 - 0.591422x + \frac{0.002743}{x}$$

$$\frac{N - N_{\min}}{N + 1} = 0.545827 - 0.591422(0.00598) + \frac{0.002743}{(0.00598)} = 0.556$$

Solving the above equation for N $(N_{\min} = 10.36 -$ see Question 8.1) yields

N = 24.6 $\approx$ 25 (or 24 + re-boiler)

The actual optimal feed stage can be calculated as follows

$$\frac{N_F}{N} = \frac{N_{F,\text{ min}}}{N_{\min}}$$

$$N_F = \left(\frac{N_{F,\text{ min}}}{N_{\min}}\right) N$$

$$N_F = \left(\frac{5.97}{10.36}\right)(24.6) = 14.2 \approx 15$$

9. We have designed a special column that acts as exactly 3 equilibrium stages, operated at total reflux. We measure vapour composition leaving the top stage and the liquid composition leaving the bottom stage. The column is separating phenol (A) from *o*-cresol (B). The phenol liquid mole fraction leaving the top stage is 0.545. What is the relative volatility of phenol with respect to *o*-cresol $\left(\alpha_{AB}\right)$?

For *total reflux*, $N = N_{\min}$; hence, form the given data, we obtain

$$N_{\min} = \frac{\log\left[\dfrac{\left(\dfrac{x_A}{x_B}\right)_{\text{dist}}}{\left(\dfrac{x_A}{x_B}\right)_{\text{bot}}}\right]}{\log\alpha_{AB}}$$

$$3 = \frac{\log\left\{\dfrac{\left[\dfrac{0.545}{(1-0.545)}\right]}{\left[\dfrac{(1-0.36)}{0.36}\right]}\right\}}{\log\alpha_{AB}}$$

Solving for α_{AB} as follows

$$\log\alpha_{AB} = \frac{\log\left\{\dfrac{\left[\dfrac{0.545}{(1-0.545)}\right]}{\left[\dfrac{0.36}{(1-0.36)}\right]}\right\}}{3} = 0.103$$

gives

$$\alpha_{AB} = 10^{0.109} = 1.285$$

10. A column with 29 equilibrium stages and a partial reboiler is being operated at total reflux to separate the mixture of ethylene dibromide (A) and propylene dibromide (B). Ethylene dibromide is more volatile, and the relative volatility, α_{AB} , is 1.30. The concentration of A in the distillate is 98.4 mol%. The column has a total condenser and saturated liquid reflux, and CMO is valid. Use the Fenske equation to estimate the composition of the bottom product

Once again, as this is the *total reflux* operation, $N = N_{\min}$

Thus, we can perform the following calculations

$$N_{\min} = \frac{\log\left[\dfrac{\left(\dfrac{x_A}{x_B}\right)_{\text{dist}}}{\left(\dfrac{x_A}{x_B}\right)_{\text{bot}}}\right]}{\log\alpha_{\text{AB}}} = \frac{\log\left\{\dfrac{\left[\dfrac{x_A}{(1-x_A)}\right]_{\text{dist}}}{\left[\dfrac{(1-x_B)}{x_B}\right]_{\text{bot}}}\right\}}{\log\alpha_{\text{AB}}}$$

$$29+1=30=\frac{\log\left\{\dfrac{\left[\dfrac{0.984}{(1-0.984)}\right]}{\left[\dfrac{(1-x_B)}{x_B}\right]_{\text{bot}}}\right\}}{\log(1.30)}$$

$$\log\left\{\frac{\left[\dfrac{0.984}{(1-0.984)}\right]}{\left[\dfrac{(1-x_B)}{x_B}\right]_{\text{bot}}}\right\}=30\times\log(1.30)$$

$$\log\left[\frac{0.984}{(1-0.984)}\right]-\log\left[\frac{(1-x_B)}{x_B}\right]_{\text{bot}}=3.418$$

$$\log\left[\frac{(1-x_B)}{x_B}\right]_{\text{bot}}=\log\left[\frac{0.984}{(1-0.984)}\right]-3.418=-1.629$$

$$\left[\frac{(1-x_B)}{x_B}\right]_{\text{bot}}=10^{(-1.629)}=0.0235$$

Solving for $x_{B,\text{ bot}}$ yields

$$x_{B,\text{ bot}}=0.977$$

Thus, the compositions of A and B of the bottom product is

- $x_A=1-x_B=1-0.977=0.023$
- $x_B=0.977$

11. We are separating a mixture of ethanol (A) and *n*-propanol (B), in which ethanol is more volatile. The relative volatility, α_{AB}, is 2.10. The feed flow rate is 1,000 kmol/h. The feed is with 60 mol% ethanol and of saturated vapour. We desire an ethanol concentration in the distillate of 99% and in the bottom product of 0.8%. Reflux is a saturated liquid. The column has 30 stages. Use the Fenske-Underwood-Gilliland techniques to determine

11.1) the number of stages at the total reflux condition

$$N_{\min} = \frac{\log\left[\frac{\left(\frac{x_A}{x_B}\right)_{\text{dist}}}{\left(\frac{x_A}{x_B}\right)_{\text{bot}}}\right]}{\log\alpha_{\text{AB}}} = \frac{\log\left\{\frac{\left[\frac{x_A}{(1-x_A)}\right]_{\text{dist}}}{\left[\frac{x_A}{(1-x_A)}\right]_{\text{bot}}}\right\}}{\log\alpha_{\text{AB}}} = \frac{\log\left\{\frac{\left[\frac{0.99}{(1-0.99)}\right]}{\left[\frac{0.008}{(1-0.008)}\right]}\right\}}{\log(2.10)} = 12.69$$

11.2) $\left(\frac{L}{D}\right)_{\min}$

Let *n*-propanol (B) a reference substance; thus

- $\alpha_A = \alpha_{AB} = \frac{K_A}{K_B} = 2.10$
- $\alpha_B = \alpha_{BB} = \frac{K_B}{K_B} = 1.00$

(note that *A* = ethanol)

Since the feed is a saturated vapour, $\Delta V_{feed} = F = 1{,}000$ kmol/h

Hence, by substituting corresponding numerical values into the following equation:

$$\Delta V_{feed} = \sum \frac{\alpha_i F z_i}{(\alpha_i - \phi)} = \frac{\alpha_A F z_A}{(\alpha_A - \phi)} + \frac{\alpha_B F z_B}{(\alpha_B - \phi)}$$

we obtain

$$1{,}000 = \frac{\alpha_A F z_A}{(\alpha_A - \phi)} + \frac{\alpha_B F z_B}{(\alpha_B - \phi)} = \frac{(2.10)(1{,}000)\left(\frac{60}{100}\right)}{(2.10-\phi)} + \frac{(1.00)(1{,}000)\left(\frac{40}{100}\right)}{(1.00-\phi)}$$

Solving the above equation for ϕ yields

$$\phi = 1.44$$

We have learned that

$$D = \left(\frac{z - x_{\text{bot}}}{x_{\text{dist}} - x_{\text{bot}}}\right)F$$

By using the data of ethanol for calculating the molar flow rate of the distillate (D), we obtain

$$D = \left[\frac{0.60-0.008}{0.99-0.008}\right](1{,}000) = 602.85 \text{ kmol/h}$$

Thus,

$$V_{\min} = \sum \frac{\alpha_i D x_{i,\text{ dist}}}{(\alpha_i - \phi)} = \frac{\alpha_A D x_{A,\text{ dist}}}{(\alpha_A - \phi)} + \frac{\alpha_B D x_{B,\text{ dist}}}{(\alpha_B - \phi)}$$

$$= \frac{(2.10)(602.85)(0.99)}{(2.10 - 1.44)} + \frac{(1.00)(602.85)(1 - 0.99)}{(1.00 - 1.44)}$$

$$V_{\min} = 1{,}885.28 \text{ kmol/h}$$

Since $V_{\min} = L_{\min} + D$, we obtain

$$L_{\min} = V_{\min} - D = 1{,}885.28 - 602.85 = 1{,}282.43 \text{ kmol/h}$$

Accordingly, the minimum reflux ratio $\left(\frac{L}{D}\right)_{\min}$ is

$$\frac{L_{\min}}{D} = \frac{1{,}282.43}{602.85} = 2.13$$

11.3) actual $\frac{L}{D}$

Since $N = 30$ (given) and $N_{\min} = 12.69$ (from 11.1), the value of the ordinate can be calculated as follows

$$y = \frac{N - N_{\min}}{N + 1} = \frac{30 - 12.69}{30 + 1} = 0.558$$

Reading from the Gilliland correlation chart at $\frac{N - N_{\min}}{N + 1} = 0.558$ results in $x =$ $\frac{(L/D) - (L/D)_{\min}}{(L/D) + 1} \approx 0.8$

Since $(L/D)_{\min}$ = 2.13 (from 11.2), the actual reflux ratio or actual $\frac{L}{D}$ can be solved as follows

$$\frac{(L/D) - (L/D)_{\min}}{(L/D) + 1} \approx 0.8$$

$$\frac{(L/D) - (2.13)}{(L/D) + 1} \approx 0.8$$

$$\text{actual } \frac{L}{D} \approx 2.4$$

Note that

$$\frac{\text{actual } \dfrac{L}{D}}{\left(\dfrac{L}{D}\right)_{\text{min}}} = \frac{2.4}{2.13} = 1.13$$

or

$$\frac{L}{D} = 1.13\left(\frac{L}{D}\right)_{\text{min}}$$