AE 335 Separation Processes

(by PTS)

Solutions to Problem Set 5

(Batch Distillation)

1. A simple batch distillation with a single equilibrium stage is employed to separate methanol from water. The feed charged to the still pot is 100 moles with 75 mol% of methanol. We desire a final bottom product concentration of 55 mol% methanol. Find the amount of the distillate collected, the amount of material remained in the still pot, and the average concentration of the distillate. The system's pressure is 1 atm; so, the equilibrium data of the methanol-water mixture given in Problem Set 1 can be used.

Equilibrium data of the methanol-water mixture are as summarised in the following Table:

Methanol liquid (mol%) (100*x*)	**Methanol vapour (mol%) (100*y*)**
0	0
2.0	13.4
4.0	23.0
6.0	30.4
8.0	36.5
10.0	41.8
15.0	51.7
20.0	57.9
30.0	66.5
40.0	72.9
50.0	77.9
60.0	82.5
70.0	87.0
80.0	91.5
90.0	95.8
95.0	97.9
100.0	100.0

The given data/information are as follows

- F = 100 kmol
- $x_F = 0.55$
- $x_{W,\text{ final}} = 0.75$

The amount of liquid remained in the still pot $\left(W_{\text{final}}\right)$ can be computed using the Rayleigh equation:

$$W_{\text{final}} = F\exp\left(-\int_{x_{W,\text{ final}}}^{x_F} \frac{dx}{y-x}\right)$$

The term $\int_{x_{W,\text{ final}}}^{x_F} \frac{dx}{y-x}$ in this Question is, in fact, $\int_{0.55}^{0.75} \frac{dx}{y-x}$, which can be obtained by determining the area under the curve of the plot between x and $\frac{1}{y-x}$ from $x = 0.55$ to $x = 0.75$, as summarised in the following Table:

x	***y***	**1/(*y* – *x*)**
0.50	0.779	1/(0.779 – 0.50) = 3.58
0.60	0.825	4.44
0.70	0.870	5.88
0.80	0.915	8.70

The area under the curve of the plot of x *vs* $\frac{1}{y-x}$ from $x = 0.55$ to $x = 0.75$ using the data in the Table above is found to be 1.056 (try drawing a graph and determining the area under the curve yourself)

Thus,

$$W_{\text{final}} = F\exp\left(-\int_{x_{W,\text{ final}}}^{x_F} \frac{dx}{y-x}\right)$$
$$= (100)\exp\left(-\int_{0.55}^{0.75} \frac{dx}{y-x}\right)$$
$$= (100)\exp(-1.056)$$
$$W_{\text{final}} = 34.8 \text{ kmol}$$

From the equation: $F = W_{\text{final}} + D_{\text{total}}$, we can calculate the value of the total amount of the distillate (D_{total}) as follows

$$D_{\text{total}} = F - W_{\text{final}} = 100 - 34.8 = 65.2 \text{ kmol}$$

The average concentration of the distillate $(x_{D,\text{ avg}})$ can be computed as follows

$$x_{D,\text{ avg}} = \frac{Fx_F - W_{\text{final}}x_{W,\text{ final}}}{D_{\text{total}}} = \frac{(100)\left(\frac{75}{100}\right) - (34.8)\left(\frac{55}{100}\right)}{65.2} = 0.857$$

2. We wish to use a distillation system of a still pot plus a column with *one* equilibrium stage to separate a mixture of methanol and water. A total condenser is used. The feed is 1 kmol with 57 mol% methanol. The desired a final bottom product concentration is 15 mol% methanol. The system's pressure is 101.3 kPa. The reflux is a saturated liquid, and $\frac{L}{D}$ is constant at 1.85. Find W_{final}, D_{total}, and $x_{D,\text{ avg}}$.

The given data/information are as follows

- F = 1 kmol
- $x_F = 0.57$
- $x_{W,\text{ final}} = 0.15$

In this Question, the Rayleigh equation is written as follows

$$W_{\text{final}} = F\exp\left(-\int_{x_{W,\text{ final}}}^{x_F} \frac{dx_W}{x_D - x_W}\right)$$

To obtain the values of $\dfrac{1}{x_D - x_W}$ at various values of x_D and the corresponding x_W, we have to do the following:

1) Draw an operating line for the selected value of x_D
2) Step off stages from the point $y = x_D$ on the $y = x$ line for a given number of stages
3) Determine the value of x_W from the graph
4) Calculate the value of $\dfrac{1}{x_D - x_W}$

It is given that $\frac{L}{D} = 1.85$; thus,

$$\frac{L}{V} = \frac{\frac{L}{D}}{1+\frac{L}{D}} = \frac{1.85}{1+1.85} = 0.65$$

and the operating line:

$$y = \frac{L}{V}x + \left(1-\frac{L}{V}\right)x_D$$

for this Question can be written as follows

$$y = 0.65x + (1-0.65)x_D = 0.65x + 0.35x_D$$

Since the system comprises a still pot and the column with 1 stage, the total number of stages (n) is 2

Hence, for each value of x_D, we draw the operating line, step off stages for 2 stages, and determine the value of x_W

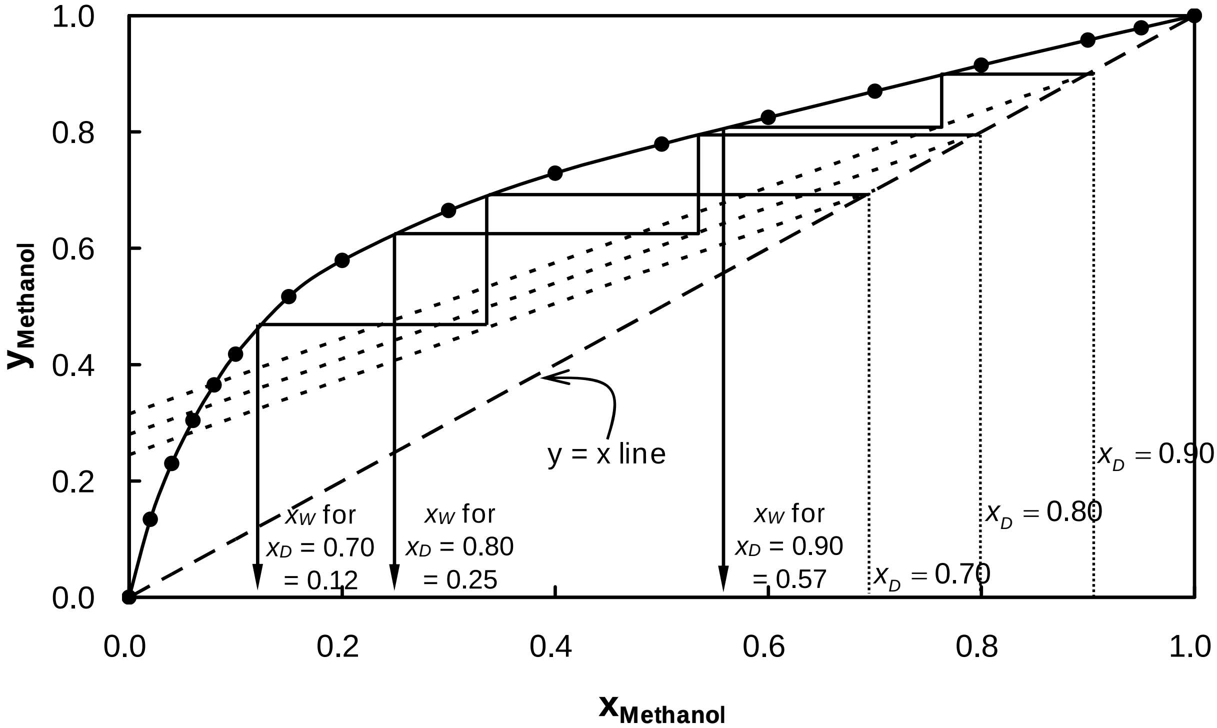
The plot, as shown on the next Page, illustrates how to step off stages and determine the value of x_W for the selected values of x_D of 0.70, 0.80, and 0.90

From the graph, we obtain the following

- For x_D = 0.70, x_W = 0.12
- For x_D = 0.80, x_W = 0.25
- For x_D = 0.90, x_W = 0.57

Additionally (but not on the graph), we also obtain

- For x_D = 0.75, x_W = 0.19
- For x_D = 0.85, x_W = 0.39



(note that the **slopes** for all cases are **constant**)

Thus, the value of $\frac{1}{x_D - x_W}$ for each x_D and corresponding x_W can be summarised in the following the Table:

x_D	x_W	$1/(x_D - x_W)$
0.70	0.12	1/(0.70 – 0.12) = 1.72
0.75	0.19	1.79
0.80	0.25	1.82
0.85	0.39	2.17
0.90	0.57	3.03

The under the curve of the plot of x_W *vs* $\frac{1}{x_D - x_W}$ from $x_W = 0.15$ to $x_W = x_F = 0.57$ is found to be 0.926 (try drawing a graph and determining the area under the curve yourself)

Hence,

$$W_{\text{final}} = F \exp\left(-\int_{x_{W,\text{ final}}}^{x_F} \frac{dx_W}{y - x} \right)$$

$$= (1) \exp\left(-\int_{0.15}^{0.57} \frac{dx_W}{x_D - x_W} \right)$$

$$= (100) \exp(-0.926)$$

$$W_{\text{final}} = 0.396 \text{ kmol}$$

Then, we can calculate the value of D_{total} as follows

$$D_{\text{total}} = F - W_{\text{final}} = 1 - 0.396 = 0.604 \text{ kmol}$$

The average concentration of the distillate $\left(x_{D,\text{ avg}}\right)$ can be computed as follows

$$x_{D,\text{ avg}} = \frac{Fx_F - W_{\text{final}}x_{W,\text{ final}}}{D_{\text{total}}} = \frac{(1)\left(\frac{57}{100}\right) - (0.396)\left(\frac{15}{100}\right)}{0.604} = 0.845$$

3. We wish to employ a normal batch distillation for the mixture of methanol and water. The system comprises a still pot and a column with 2 equilibrium stages. The column has a total condenser, and the reflux is a saturated liquid. The column is operating with a *varying* reflux ratio, but x_D is held *constant*. The initial feed charged into the still pot is 10 kmol with 40 mol% methanol. The desired final concentration in the still pot is 8 mol% methanol and the desired distillate concentration is 85 mol% methanol. The system's pressure is 1 atm, and CMO is valid.

3.1) What initial external reflux ratio $\left(\frac{L}{D}\right)$ must be used?

3.2) What final external reflux ratio must be used?

3.3) How much distillate product is withdrawn, and what is the final amount of material left in the still pot?

The given data/information are as follows

- F = 10 kmol
- $x_F = 0.40$
- $x_{W,\text{ final}} = 0.08$
- $x_{D,\text{ avg}} = 0.85$

In this Question, the value of x_D is fixed at 0.85, and the system is operated by changing the reflux ratio $\left(\frac{L}{D}\right)$

Hence, the **slope** of the **operating line** is **NOT constant**

To obtain the value of the *initial* external reflux ratio, we have to use a trial & error technique by

1) guessing the value of the reflux ratio $\left(\frac{L}{D}\right)$ and thus computing value of the slope of the operating line $\left(\frac{L}{V}\right)$, or we can make a guess for the value of $\frac{L}{V}$ directly
2) drawing the operating line with the slope obtained from 1
3) trying stepping off stages from the point where $y = x_F$ on the $y = x$ line to the point where $y = x_D$ on the $y = x$ line
4) If the guessed slope (from 1) is correct, the stages must be equal to that specified by the problem
5) If NOT, or the guessed slope (from 1) does not give the number of stages equals that specified by the problem statement, we have to make a new guess

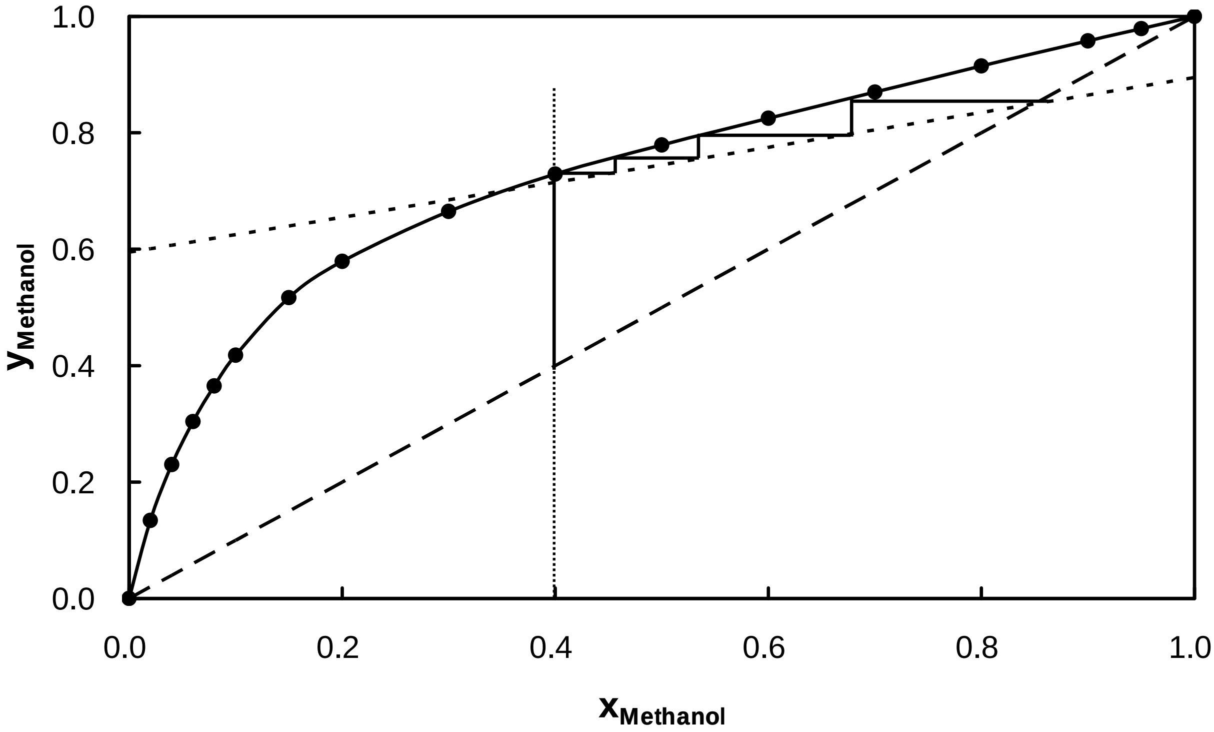
In this Question, **the total number of stages is 3** (a still pot with a column with 2 stages)

We start guessing the slope $\left(\frac{L}{V}\right)$ of 0.30; hence, the operating line is

$$y = \frac{L}{V}x + \left(1 - \frac{L}{V}\right)x_D$$
$$= 0.30x + (1 - 0.30)(0.85)$$
$$y = 0.30x + 0.595$$

(note that the value of x_D is fixed at 0.85 as specified in the problem statement)

Drawing the operating line from the equation: $y = 0.30x + 0.595$ on the McCabe-Thiele diagram and stepping off stages, from the point where $y = x_F = 0.40$ to the point where $y = x_D = 0.85$, yields the number of stages of **~4**, which is **not 3**



So, we need a new guess

By doing trial & error to find the correct value of the slope, we found that the appropriate **slope** value that gives the number of stages of 3 is **~0.32** (try doing it yourself)

Since $\frac{L}{D} = \frac{\frac{L}{V}}{1 - \frac{L}{V}}$, the appropriate *initial* external reflux ratio is

$$\frac{L}{D} = \frac{0.32}{1 - 0.32} = 0.47$$

For the *final* external reflux ratio, the procedure is still the same, but it starts from the point where $y = x_{W,\text{ final}}$ on the $y = x$ line to the point where $y = x_D$ on the $y = x$ line

Once again, we start guessing the slope $\left(\frac{L}{V}\right)$, but, this time, the guessed slope of 0.85; hence, the operating line is

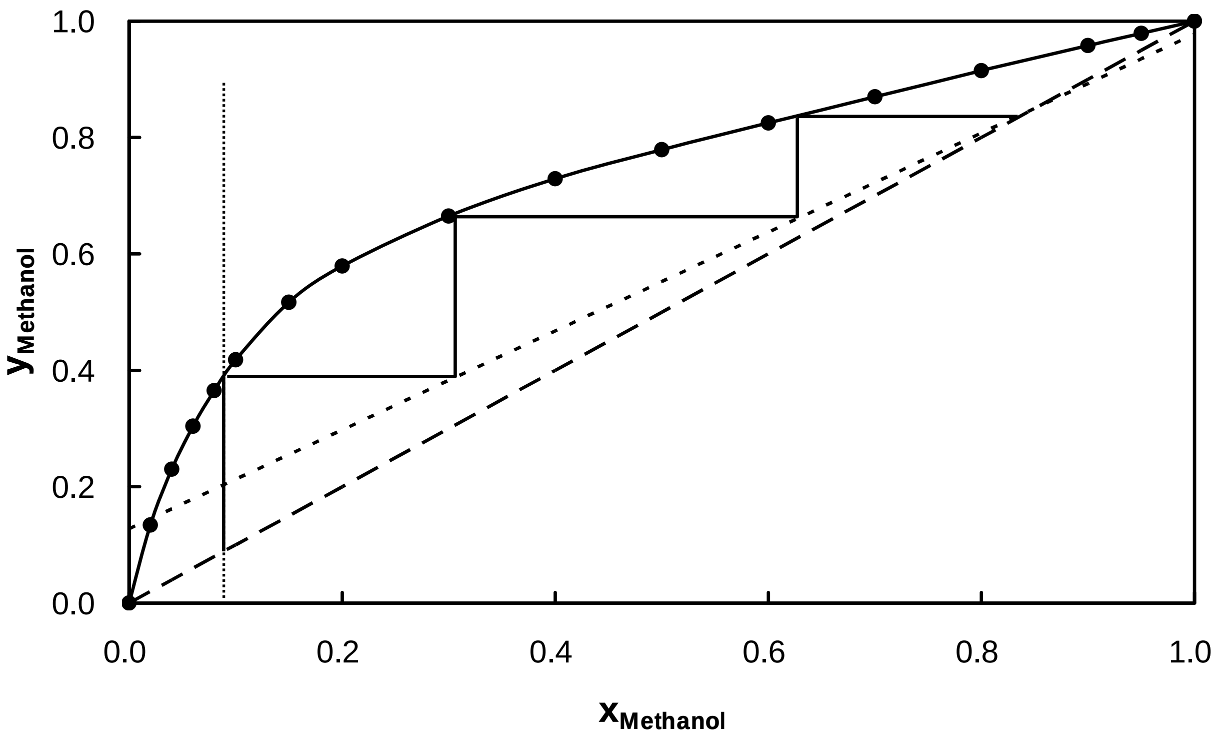
$$y = \frac{L}{V}x + \left(1 - \frac{L}{V}\right)x_D$$

$$= 0.85x + (1 - 0.85)(0.85)$$

$$y = 0.85x + 0.1275$$

(note again that the value of x_D is fixed at 0.85)

Drawing the operating line from the equation above on the McCabe-Thiele diagram and stepping off stages, from the point where $y = x_{W,\text{ final}} = 0.08$ to the point where $y = x_D = 0.85$, yields the number of stages of **~3**



So, the guessed slope (*i.e.* $\frac{L}{V} = 0.85$) seems to be correct

Thus, the *final* external reflux ratio is

$$\frac{L}{D} = \frac{\frac{L}{V}}{1 - \frac{L}{V}} = \frac{0.85}{1 - 0.85} = 5.67$$

Solving the overall and species balances simultaneously

$$D_{\text{total}} + W_{\text{final}} = F$$
$$D_{\text{total}} + W_{\text{final}} = 10$$

and

$$x_{D,\text{ avg}} D_{\text{total}} + x_{W,\text{ final}} W_{\text{final}} = x_F F$$
$$(0.85) D_{\text{total}} + (0.08) W_{\text{final}} = (0.40)(10)$$

gives

$$W_{\text{final}} = 5.84 \text{ kmol}$$
$$D_{\text{total}} = 4.16 \text{ kmol}$$

4. A mixture of 62 mol% methanol and the remaining water is distilled using a batch distillation. The batch distillation system comprises a still pot and a column with one equilibrium stage. The feed is 3 kmol. The system operates at the *constant* distillate concentration (x_D) of 85 mol% methanol. The desired final still pot concentration is 45 mol% methanol. The reflux is a saturated liquid. Assume that CMO is valid
   4.1) Find D_{total} and W_{final}
   4.2) Find the final value of the external reflux ratio $\left(\frac{L}{D}\right)$

The given data/information are as follows

- F = 3 kmol
- $x_F = 0.62$
- $x_{W,\text{ final}} = 0.45$
- $x_{D,\text{ avg}} = 0.85$

The procedure is similar to that in Question 3; so, try doing it yourself

The answers are:

- The *final* external reflux ratio is found to be ~0.93 (and the correct guessed slope of the operating line is ~0.48)
- $W_{\text{final}} = 1.725$ kmol and $D_{\text{total}} = 1.275$ kmol