

# AE 335 Separation Processes

## (by PTS)

### Solutions to Problem Set 6

#### (Absorption & Stripping)

1. We are absorbing hydrogen sulphide ( $\text{H}_2\text{S}$ ) in biogas at  $15^\circ\text{C}$  into water. The entering water is pure. The feed gas contains 0.0012 mole fraction  $\text{H}_2\text{S}$  and the remaining  $\text{CH}_4$ , and we want to remove 97 mol% of  $\text{H}_2\text{S}$  in biogas into water. The gas flow rate is 10 kmol/h, while the liquid flow rate is 2,000 kmol/h. Total pressure is 2.5 atm. The equilibrium data is

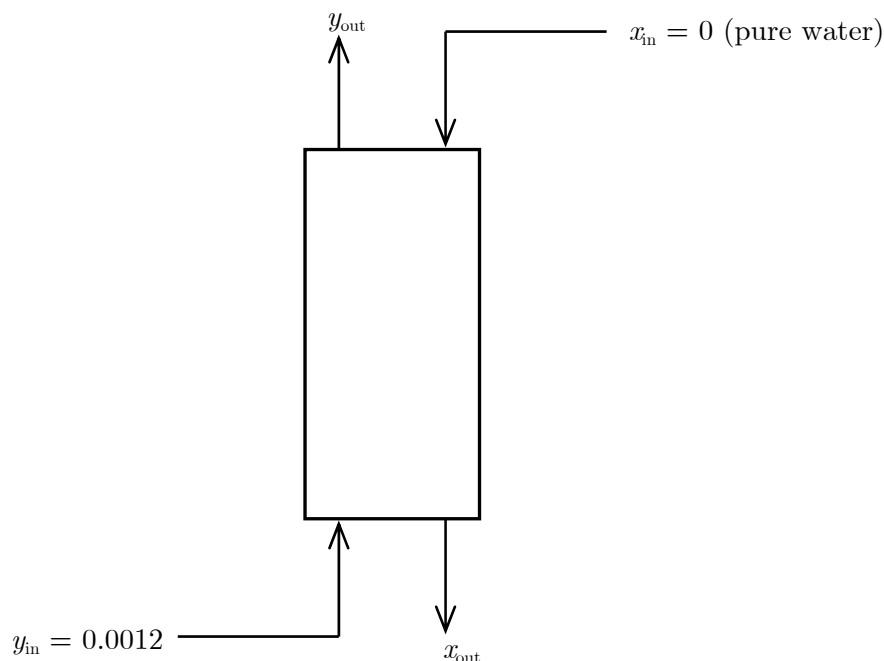
$$\text{Partial pressure of } \text{H}_2\text{S (atm)} = (\text{Henry's constant})x$$

where Henry's constant of  $\text{H}_2\text{S}$  in water at  $15^\circ\text{C}$  is 423 atm/(mole fraction) and  $x$  is mole fraction of  $\text{H}_2\text{S}$ .

- 1.1) Calculate the outlet gas and liquid mole fraction of  $\text{H}_2\text{S}$
- 1.2) Calculate the number of equilibrium stages required, using a McCabe-Thiele diagram
- 1.3) If actual  $\frac{L}{V} = m \left( \frac{L}{V} \right)_{\min}$ , find the value of  $m$  (a multiplier)

Note that, in this Question, since the concentration of the solute (*i.e.*  $\text{H}_2\text{S}$ ) is relatively low, it is reasonable to assume that the flow rate of the liquid and gas phases are constant, even though the solute is moving from the gas phase to the liquid phase, and by assuming such that, the **mole fractions** for the compositions in the **gas** and **liquid** phases (*i.e.*  $y$  and  $x$ ) are used instead of  $Y$  and  $X$ , respectively

The schematic diagram of this absorption process is as shown below



From the given data/information, we obtain the following:

- $y_{in} = 0.0012$
- $x_{in} = 0$  (as *pure* water is used as a solvent)
- $G$  or  $V = 10$  kmol/h
- $L = 2,000$  kmol/h
- The equilibrium equation is  $y = \frac{423x}{2.5} = 169.2x$

It is required that 97% of  $H_2S$  is removed; thus, the concentration of  $H_2S$  in the outlet stream is

$$y_{out} = (1 - 0.97)(0.0012) = 0.000036$$

By performing a species balance for  $H_2S$ , we obtain

$$\begin{aligned} y_{in}V + x_{in}(L) &= y_{out}V + x_{out}L \\ 0.0012(10) + 0(2,000) &= 0.000036(10) + x_{out}(2,000) \\ x_{out} &= 5.82 \times 10^{-6} \text{ or } 5.82 \text{ ppm} \end{aligned}$$

The operating line passes through two points, *i.e.*  $(x_{out}, y_{in})$  and  $(x_{in}, y_{out})$  or  $(5.82 \times 10^{-6}, 0.0012)$  and  $(0, 0.000036)$

The equilibrium line can be drawn from the equilibrium equation:  $y = 169.2x$

By drawing the operating line and the equilibrium line on the same McCabe-Thiele diagram, we can step off stages, which, in this Question, is found to be ~10.5 (note that for the absorption operation, the equilibrium line is under the operating line)

### Try doing it yourself

To obtain the operating line with minimum slope or to obtain minimum  $\frac{L}{V}$  or  $\left(\frac{L}{G}\right)_{min}$ , we have to draw the line from the point  $(x_{in}, y_{out})$  or  $(0, 0.000036)$  until the line touches the equilibrium line at the point where  $y = y_{in} = 0.0012$

At the touching point, we obtain  $x_{out, min} = 7.09 \times 10^{-6}$ , and the slope of this line or  $\left(\frac{L}{G}\right)_{min}$  can be computed as follows

$$\text{slope (min)} = \left(\frac{L}{G}\right)_{min} = \frac{y_{in} - y_{out}}{x_{out, min} - x_{in}} = \frac{0.0012 - 0.000036}{7.09 \times 10^{-6} - 0} = 164.1$$

The slope of the actual operating line or  $\frac{L}{G}$ , which passes through the points  $(x_{out}, y_{in})$  and  $(x_{in}, y_{out})$  or  $(5.82 \times 10^{-6}, 0.0012)$  and  $(0, 0.000036)$ , can be calculated as follows

$$\text{slope (actual)} = \left(\frac{L}{G}\right)_{actual} = \frac{y_{in} - y_{out}}{x_{out} - x_{in}} = \frac{0.0012 - 0.000036}{5.82 \times 10^{-6} - 0} = 200$$

or it can alternatively be computed as follows

$$\text{actual} \left( \frac{L}{G} \right)_{\text{actual}} = \frac{\text{Liquid flow rate}}{\text{Gas flow rate}} = \frac{2,000 \text{ kmol}}{10 \text{ kmol}} = 200$$

Hence, the value of multiplier ( $m$ ) is

$$m = \frac{\left( \frac{L}{G} \right)_{\text{actual}}}{\left( \frac{L}{G} \right)_{\text{min}}} = \frac{200}{164} = 1.22$$

2. We wish to design a stripping column to remove carbon dioxide (CO<sub>2</sub>) from water. This can be done by heating the water + CO<sub>2</sub> mixture and passing it counter-currently with a nitrogen stream in a stripper. The operation is isothermal and isobaric at 60 °C and 1 atm, respectively. The water contains  $9.2 \times 10^{-6}$  mole fraction CO<sub>2</sub> and flows at 100,000 lb<sub>m</sub>/h. Nitrogen enters the column as pure N<sub>2</sub> and flows at 2,500 ft<sup>3</sup>/h. Nitrogen is also at 1 atm and 60 °C. We desire an outlet water concentration of  $2 \times 10^{-7}$  mole fraction CO<sub>2</sub>. Assume that N<sub>2</sub> is not dissolved in water and that water is not evaporated. The Henry's constant for CO<sub>2</sub> in water at 60 °C is 3,410 atm/(mole fraction). Find the number of equilibrium stages required.

By using the equation of state for ideal gases:

$$PV = nRT$$

the volume of 1 lb-mol of N<sub>2</sub> at 1 atm (or 14.7 psi) and 60 °C (140 °F or 140+460 = 600 °R) can be calculated as ~438 ft<sup>3</sup> (try computing this value yourself, using the gas constant,  $R$ , of 0.7302 ft<sup>3</sup>·atm/°R·lb-mol or 10.732 ft<sup>3</sup>·psi/°R·lb-mol)

Hence, the molar flow rate of N<sub>2</sub> for the volumetric flow rate of 2,500 ft<sup>3</sup>/h is

$$\frac{2,500 \text{ ft}^3/\text{h}}{438 \text{ ft}^3/\text{lb-mol}} = 5.71 \text{ lb-mol/h}$$

MW of water is ~18 lb/lb-mol

Thus, the molar flow rate of water (+ CO<sub>2</sub>) for the mass flow rate of 100,000 lb<sub>m</sub>/h is

$$\frac{100,000}{18} = 5,560 \text{ lb-mol/h}$$

(Note that, even though water is NOT pure, but the amount of CO<sub>2</sub> is extremely low, it is safe or reasonable to assume that the flow rate of the water + CO<sub>2</sub> mixture is about the same as the flow rate of pure water)

Thus, the actual  $\frac{L}{V}$  is

$$\frac{5,560}{5.71} = 973.7$$

It is given, in the problem statement, that

- $x_{in} = 9.2 \times 10^{-6}$
- $x_{out} = 2 \times 10^{-7}$
- $y_{in} = 0$  (as pure nitrogen, N<sub>2</sub>, is used as a stripping gas)

Performing a species balance for CO<sub>2</sub> gives

$$y_{out}V + x_{out}L = y_{in}V + x_{in}L$$

$$y_{out}(5.72) + (2 \times 10^{-7})(5,560) = (0)(5.72) + (9.2 \times 10^{-6})(5,560)$$

$$y_{out} \approx 0.00875$$

The operating line passes through two points: ( $x_{out}$ ,  $y_{in}$ ) and ( $x_{in}$ ,  $y_{out}$ ) or ( $2 \times 10^{-7}$ , 0) and ( $9.2 \times 10^{-6}$ , 0.00875)

The equilibrium line can be drawn from the equilibrium equation as follows:

$$y = \frac{3,410x}{1} = 3,410x$$

By drawing the operating line and the equilibrium line on the same McCabe-Thiele diagram, we can step off stages, which, in this Question, is found to be ~3 (note, since this is the stripping operation, the equilibrium line is above the operating line)

3. A stripping tower with 4 equilibrium stages is being used to remove ammonia from waste water using air as the stripping gas. The operation is at 80 °F and 1 atm. The inlet air is pure air, and the inlet waste water contains 0.02 mole fraction ammonia. The column operates at  $L/V$  of 0.65. The equilibrium data in mole fraction is given as  $y = 1.414x$ . Find the outlet concentrations.

It is given that:

- $N = 4$
- $m = 1.414$
- $\frac{L}{V} = 0.65$
- $y_{in} = 0$  (as pure air is used as a stripping gas) – note that  $y_{in} = y_{N+1}$
- $y_{out}^* = mx_{in}$  :  $y_{out}^* = (1.414)(0.02) = 0.02828$   
(this is because, at equilibrium, the liquid phase that goes in is in equilibrium with the gas phase that goes out – note that  $y_{out}^* = y_1^*$  and  $x_{in} = x_o$ )

Actually, this Question can be solved in the same manner as Question 2, but, in this Question, a Kremser equation is employed to solve for the outlet concentration of the liquid phase ( $x_{out}$ ), in order to illustrate how Kremser equation can be used to solve this kind of problem

Since the given data are for the concentrations in the **gas** phase,  $y$ , the following Kremser equation is chosen:

$$\frac{y_{out} - y_{out}^*}{y_{in} - y_{out}^*} = \frac{1 - \frac{L}{mV}}{1 - \left(\frac{L}{mV}\right)^{N+1}}$$

Substituting given corresponding numerical values into the above Kremser equation and solving for  $y_{out}$  yields  $y_{out} = 0.0127$

It is noteworthy that, for the stripping operation in this Question, for the same  $x_{in}$  ( $= 0.02$ ), the value of  $y_{out}^*$  (from the equilibrium line/equation  $= 0.02828$ ) is higher than the value of  $y_{out}$  (from the operating line/equation  $= 0.0127$ ) – this confirms that, for the stripping operation, the equilibrium line is above the operating line

Performing a species balance for ammonia yields

$$\begin{aligned} y_{out}V + x_{out}L &= y_{in}V + x_{in}L \\ (0.0127)L + x_{out}V &= (0)V + (0.02)L \\ x_{out} &= (0.0127)\frac{L}{V} + (0)\frac{L}{V} + (0.02) = 4.93 \times 10^{-4} \end{aligned}$$

4. An absorption column for laboratory use has been carefully constructed so that it has exactly 4 equilibrium stages and is being used to measure equilibrium data. Water is used as the solvent to absorb ammonia from air. The system operates isothermally at 80 °F and isobarically at 1 atm. The inlet water is pure distilled water. The ratio of  $L/V$  is 1.2, the inlet gas concentration is 0.01 mole fraction ammonia, and the measured outlet gas concentration is 0.0027 mole fraction ammonia. Assuming the equilibrium is of the equation  $y = mx$ , determine the value of  $m$ .

It is given that

- $N = 4$
- $\frac{L}{V} = 1.2$
- $y_{N+1} = 0.01$
- $y_1 = 0.0027$
- $x_o = 0$  (as pure distilled water is used as a solvent); thus,  
 $y_1^* = mx_o : y_1^* = m(0) = 0$

In this Question, since we cannot draw an equilibrium line [because we have not known the value of  $m$  (or the slope of the equilibrium line) yet], it would be more convenient to use a Kremser equation solve this question – *i.e.* to solve for the value of  $m$  (note that, since the given data are for the compositions of **gas** phase,  $y$ , the following Kremser equation is chosen):

$$\frac{y_1 - y_1^*}{y_{N+1} - y_1^*} = \frac{1 - \frac{L}{mV}}{1 - \left(\frac{L}{mV}\right)^{N+1}}$$

Substituting corresponding given numerical values into the above Kremser equation (note that,  $x_o = x_{in}$ ,  $y_1 = y_{out}$ ,  $y_{N+1} = y_{in}$ , and  $y_1^* = y_{out}^*$ , which can be computed from the equation:  $y_1^* = mx_o$ )

gives

$$\frac{0.0027 - 0}{0.01 - 0} = \frac{1 - \frac{1.2}{m}}{1 - \left(\frac{1.2}{m}\right)^{4+1}}$$

Solving for  $m$  yields

$$m \approx 1.415$$

5. We wish to strip  $\text{CO}_2$  out of water at  $20^\circ\text{C}$  and 2 atm using a staged, counter-current stripper. The liquid flow rate is 100 kmol/h of water, and the initial  $\text{CO}_2$  mole fraction in water is 0.00005. The inlet air stream contains no  $\text{CO}_2$ . It is desired to obtain 98.4% removal of  $\text{CO}_2$  from water. The Henry's constant for  $\text{CO}_2$  in water at  $20^\circ\text{C}$  is 1,420 atm.

5.1) Find the outlet  $\text{CO}_2$  mole fraction in water

5.2) Find  $V_{\min}$

5.3) If there are 7 equilibrium stages, find  $V$  and the outlet mole fraction  $\text{CO}_2$  in water

From the given data, the equilibrium equation for this Question is

$$y = \frac{1,420x}{2} = 710x$$

It is required that 98.4% of  $\text{CO}_2$  be removed, and the initial concentration of  $\text{CO}_2$  is 0.00005

Thus, the outlet concentration of  $\text{CO}_2$  in water is

$$(1 - 0.984)(0.00005) = 8 \times 10^{-7}$$

Draw the equilibrium line from the equilibrium equation (*i.e.*  $y = 710x$ ) on the McCabe-Thiele diagram

Since the operating line must pass through the  $(x_{out}, y_{in})$  point, the *maximum* slope of the operating line or  $\left(\frac{L}{V}\right)_{\max}$  can be obtained by drawing the line from the point  $(x_{out}, y_{in}) = (8 \times 10^{-7}, 0)$  until it touches the equilibrium line at the point where  $x = x_{in} = 0.00005$  (note that  $y_{in} = 0$  as the inlet air, a stripping gas, contains no  $\text{CO}_2$ , a solute)

### Try drawing the equilibrium line and the maximum operating line yourself

At the touching point, the value of  $y = y_{out, \max} = 0.0355$

Then, the maximum slope or  $\left(\frac{L}{V}\right)_{\max}$  of this system can then be computed as follows

$$\left(\frac{L}{V}\right)_{\max} = \frac{y_{in} - y_{out, \max}}{x_{out} - x_{in}} = \frac{0 - 0.0355}{8 \times 10^{-7} - 0.00005} = 721.5$$

For  $L = 100$  kmol/h, the value of  $V_{\min}$  can be calculated as follows

$$\left(\frac{L}{V}\right)_{\max} = 721.5$$

$$V_{\min} = \frac{L}{\left(\frac{L}{V}\right)_{\max}} = \frac{100}{721.5} = 0.14 \text{ kmol/h}$$

To solve for Question 5.3, several operating lines must be drawn until we obtain the operating line that enables the column to have 7 stages and the final stage that passes through the point where  $x = x_{in} = 0.00005$

### Try doing it yourself

The actual  $y_{out}$  is found to be 0.0224, and the actual  $\frac{L}{V}$  can be computed as follows

$$\left(\frac{L}{V}\right)_{\text{actual}} = \frac{y_{in} - y_{out}}{x_{out} - x_{in}} = \frac{0 - 0.0224}{8 \times 10^{-7} - 0.00005} = 455.3$$

The value of **actual V** can, then, be computed as follows

$$V_{\text{actual}} = \frac{L}{\left(\frac{L}{V}\right)_{\text{actual}}} = \frac{100}{455.3} = 0.22 \text{ kmol/h}$$

6. We wish to absorb ammonia from air into water. The equilibrium data is given as  $y = 1.414x$  in mole fraction. The counter-current column has 3 equilibrium stages. The entering air stream has a total flow rate of 10 kmol/h and is with 0.0083 mole fraction  $\text{NH}_3$ . The inlet water stream contains 0.0002 mole fraction  $\text{NH}_3$ . We desire an outlet gas concentration of 0.0005 mole fraction  $\text{NH}_3$ . Find the required liquid flow rate,  $L$ .

By using the following Kremser equation:

$$\frac{y_{N+1} - y_1^*}{y_1 - y_1^*} = \frac{1 - \left(\frac{L}{mV}\right)^{N+1}}{1 - \frac{L}{mV}}$$

for the following given data:

- $N = 3$
- $y_{N+1} = 0.0083$
- $y_1 = 0.0005$
- $x_o = 0.0002$ ; thus,  $y_1^* = mx_o$ ;  $y_1^* = 1.414(0.0002) = 0.0002828$

results in

$$\frac{0.0083 - 0.0002828}{0.0005 - 0.0002828} = \frac{1 - \left(\frac{L}{mV}\right)^{3+1}}{1 - \frac{L}{mV}}$$

Solving for  $\frac{L}{mV}$  (by trial & error – until we obtain the value of  $\frac{L}{mV}$  that makes both sides of equation equal to each other) gives

$$\frac{L}{mV} = 2.91$$

Thus,

$$L = 2.91mV = 2.91 \times 1.414 \times 10 = 41.1 \text{ kmol/h}$$